Foreword

This Handbook is growing in size, reflecting the fact that Teichmüller theory has multiple facets and is being developed in several directions.

In this new volume, as in the preceding volumes, there are chapters that concern the fundamental theory and others that deal with more specialized developments. Some chapters treat in more detail subjects that were only briefly outlined in the preceding volumes, and others present general theories that were not treated there. The study of Teichmüller spaces cannot be dissociated from that of mapping class groups, and like in the previous volumes, a substantial part of the present volume deals with these groups.

The volume is divided into the following four parts:

- The metric and the analytic theory, 3.
- The group theory, 3.
- The algebraic topology of mapping class groups and moduli spaces.
- Teichmüller theory and mathematical physics.

The numbers that follow the titles in the first two parts indicate that there were parts in the preceding volumes that carry the same titles.

This Handbook is also a place where several fields of mathematics interact. For the present volume, one can mention the following: partial differential equations, one and several complex variables, algebraic geometry, algebraic topology, combinatorial topology, 3-manifolds, theoretical physics, and there are several others. This confluence of ideas towards a unique subject is a manifestation of the unity and harmony of mathematics

In addition to the fact of providing surveys on Teichmüller theory, several chapters in this volume contain expositions of theories and techniques that do not strictly speaking belong to Teichmüller theory, but that have been used in an essential way in the development of this theory. Such sections contribute in making this volume and the whole set of volumes of the Handbook quite self-contained. The reader who wants to learn the theory is thus spared some of the effort of searching into several books and papers in order to find the material that he needs. For instance, Chapter 4 contains an introduction to arithmetic groups and their actions on symmetric spaces, with a view towards comparisons and analogies between this theory and the theory of mapping class groups and their action on Teichmüller spaces. Chapter 5 contains an introduction to abstract simplicial complexes and their automorphisms. Chapter 9 contains a concise survey of group homology and cohomology, and an exposition of the Fox calculus, having in mind applications to the theory of the Magnus representation of the mapping class group. Chapter 10 contains an exposition of the theory of Thompson's groups in relation with Teichmüller spaces and mapping class groups. The same chapter contains a review of Penner's theory of the universal

decorated Teichmüller space and of cluster algebras. Chapter 10 and Chapter 14 contain an exposition of the dilogarithm, having in mind its use in the quantization theory of Teichmüller space and in the representation theory of mapping class groups. Chapter 11 contains a section on the intersection theory of complex varieties, as well as an introduction to the theory of characteristic classes of vector bundles, with applications to the intersection theory of the moduli space of curves and of its stable curve compactification. Chapter 13 contains an exposition of L^p-cohomology, of the intersection cohomology theory for projective algebraic varieties and of the Hodge decomposition theory for compact Kähler manifolds, with a stress on applications to Teichmüller and moduli spaces.

Finally, let us mention that several chapters in this volume contain open problems directed towards future research; in particular Chapter 4 by Ji, Chapter 5 by McCarthy and myself, Chapter 7 by Korkmaz, Chapter 8 by Habiro and Massuyeau, Chapter 9 by Sakasai, Chapter 10 by Funar, Kapoudjian and Sergiescu, and Chapter 13 by Ji and Zucker.

Up to now, sixty different authors (some of them with more than one contribution) have participated to this project, and there are other authors, working on volumes in preparation. I would like to thank them all for this fruitful cooperation which we all hope will serve generations of mathematicians.

I would like to thank once more Manfred Karbe and Vladimir Turaev for their interest and their care, and Irene Zimmermann for the seriousness of her work.

Strasbourg, April 2012 Athanase Papadopoulos