Preface

The Fourteenth International Conference on Representations of Algebras and Workshop (ICRA XIV) was held at the National Olympics Memorial Youth Center (NYC) in Tokyo, Japan, from 6 to 15 August, 2010. The ICRA XIV was attended by the remarkable large number of 230 researchers and graduate students from 23 countries of all parts of the world.

The Scientific Advisory ICRA XIV Committee consisted of R. Bautista (Morelia, Mexico), R.-O. Buchweitz (Toronto, Canada), M. C. R. Butler (Liverpool, United Kingdom), W. Crawley-Boevey (Leeds, United Kingdom), V. Dlab (Ottawa, Canada), Y. A. Drozd (Kiev, Ukraine), K. Erdmann (Oxford, United Kingdom), B. Huisgen-Zimmermann (Santa Barbara, United States), B. Keller (Paris, France), H. Lenzing (Paderborn, Germany), M.-P. Malliavin (Paris, France), H. Merklen (Sao Paulo, Brazil), J.A. de la Peña (Mexico City, Mexico), M. I. Platzeck (Bahia Blanca, Argentina), I. Reiten (Trondheim, Norway), C. M. Ringel (Bielefeld, Germany), D. Simson (Toruń, Poland), A. Skowroński (Toruń, Poland), S. O. Smalø (Trondheim, Norway), K. Yamagata (Tokyo, Japan), Y. Zhang (Beijing, China).

The Local Organizing ICRA XIV Committee was formed by K. Yamagata (Chairman), H. Asashiba, O. Iyama, S. Koshitani, I. Mori, K. Nishida, M. Sato.

We would like to thank the members of the Committees as well the leaders of research groups for the advices, help and cooperation making the ICRA XIV very successful. We are also grateful to the National Olympics Memorial Youth Center in Tokyo for the possibility to organize the ICRA XIV in this wonderful place and to the Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (B) 21340003, for a substantial financial support.

According to a tradition in the area, the ICRA XIV was divided into two parts: the Workshop and the Conference. The ICRA XIV Workshop consisted of seven mini-courses of three hours each given by M. Kashiwara (Kyoto), B. Keller (Paris), B. Leclerc (Caen), H. Lenzing (Paderborn), M. Linckelmann (Aberdeen), C. M. Ringel (Bielefeld), G. Zwara (Toruń). The ICRA XIV Conference comprised 124 talks (24 plenary talks, 100 talks in parallel sessions), among them 14 one hour plenary lectures given by R.-O. Buchweitz (Toronto), J. Chuang (London), A. Henke (Garching), O. Iyama (Nagoya), S. B. Iyengar (Nebraska), D. Kussin (Paderborn), I. Mori (Shizuoka), J. A. de la Peña (Mexico City), I. Reiten (Trondheim), J. Schröer (Bonn), A. Skowroński (Toruń), C. Xi (Beijing), Y. Yoshino (Okayama), A. Zelevinsky (Boston).

The ICRA AWARD 2010 (for outstanding work by young mathematician working in the area of representation theory of algebras) was given to Claire Amiot for her original and influential work on 2-Calabi–Yau categories and in particular her construction of generalized cluster categories associated to quivers with potential and to algebras of global dimension two.

This book contains eleven expository survey articles and two research articles on recent developments and trends in the area of representation theory of algebras and

related topics, reflecting the topics of some lectures presented during the ICRA XIV Workshop and Conference held in Tokyo.

We now briefly describe the contents of the articles.

The article by Amiot surveys development and motivations on cluster categories leading from the cluster categories of finite dimensional hereditary algebras defined by Buan, Marsh, Reineke, Reiten and Todorov as well as the stable categories of modules over preprojective algebras of Dynkin type studied by Geiss, Leclerc and Schröer to her generalized cluster categories associated to the Jacobi-finite quivers with potential and to finite dimensional algebras of global dimension at most two. In the article, the general construction of these new triangulated 2-Calabi–Yau categories and links with other related categories are presented. Moreover, several interesting applications of the generalized cluster categories in the representation theory of finite dimensional algebras are outlined.

The purpose of the article by Benson, Iyengar and Krause is to explain the recent work of the three authors on the classification of localizing and colocalizing subcategories of triangulated categories. In the article, the main attention is devoted to the stable module categories of group algebras of finite groups over algebraically closed fields of positive characteristic. A prominent role of thick subcategories of the considered triangulated categories as well as the support varieties of finitely generated modules over the group algebras of finite groups for the discussed classification problems is illuminated. Moreover, interesting applications of the authors classification of localizing subcategories are exhibited.

The aim to the article by Keller is to give an introduction to quantum dilogarithm identities as well as explain connections to Fomin–Zelevinsky theory of cluster algebras. In the first part of the article Keller explains Reineke's identities between products of quantum dilogarithm series associated with Dynkin quiver, extending the dilogarithm identities established for two quantum variables by Schützenberger, Faddeev–Volkov, and Faddeev–Kashaev. The second part of the article is devoted to similar quantum dilogarithm identifies for quivers with potential, following ideas due to Bridgeland, Fock–Goncharov, Kontsevich–Soilbelman and Nagao. Moreover, a prominent role of stability functions, Hall algebras and Jacobian algebras for proving the considered dilogarithm identities is exhibited.

The article by Leclerc explains connections between quantum loop algebras of simple complex Lie algebras, Nakajima quiver varieties and cluster algebras of Fomin–Zelevinsky. In particular, an introduction to finite dimensional representations of quantum loop algebras and Nakajima's geometric description of the irreducible q-characters in terms of graded quiver varieties is provided. The final part of the article is devoted to a recent attempt to understand the tensor structure of the category of finite dimensional representations of the quantum loop algebra of an arbitrary simple complex Lie algebra by means of cluster algebras. Here a general conjecture by Hernandez and Leclerc as well as well as its solution in a special case are discussed.

The article by Lenzing serves as a guide to the theory of coherent sheaves over weighted projective lines and applications, since their introduction in 1987 by Geigle

and the author to recent developments. In particular, a recent application of the theory to the analysis of the singularity category of triangle singularities, Kleinian and Fuchsian singularities is covered in some details. A prominent role of the stable categories over vector bundles over weighted projective lines in this analysis is exhibited. The author outlines the results of his recent joint work with Kussin, Meltzer and de la Peña as well as connections with the work by Buchweitz, Orlov, and the recent work by Kajiura, Saito and Takahashi. Further, an application of presented methods to the study of invariant subspace problem for nilpotent operators initiated by Ringel and Schmidmeier is also presented.

The main general question discussed in the article by Linckelmann is a description of finite dimensional algebras which occur as indecomposable direct factors (blocks) of group algebras of finite groups over an algebraically closed field of positive characteristic p. The second main question discussed in the article is to what extent the blocks of finite group algebras are determined by their defect groups and fusion systems. The author provides an introduction to the basic theory of blocks of finite group algebras as well as surveys old and recent results concerning the two questions raised above, invoking the block cohomology and Hochschild cohomology of blocks. In particular, prominent conjectures in block theory (finiteness conjectures, counting conjectures, structural conjectures) and their partial confirmations are presented and discussed.

The article by Malicki and Skowroński surveys old and new results on the structure and homological properties of Artin algebras whose Auslander–Reiten quiver admits a separating family of connected components. In the article, many important results on the structure of module categories of distinguished classes of Artin algebras (tilted algebras, quasitilted algebras, double tilted algebras, generalized double tilted algebras, generalized multicoil algebras) with separating families of Auslander–Reiten components, established during the last 30 years, as well as illustrating examples are presented in details. In the final part of the article, a complete description of the module categories of arbitrary Artin algebras having separating families of Auslander–Reiten components (equivalently, module categories with hearts) is presented. In particular, the generically tame Artin algebras with separating families of Auslander–Reiten components as well as their module categories are described completely.

The article by Mori discusses classification problems in the noncommutative algebraic geometry and their strong connections with classification problems of homologically nice finite dimensional algebras over a field. The author starts with two major achievements in the noncommutative algebraic geometry: the classification of quantum projective planes and the classification of noncommutative projective curves. Then an interesting new nice class of finite dimensional algebras of finite global dimension, called quasi-Fano algebras, is introduced and investigated. Further, generalizations of Artin–Schelter regular algebras and the structure of Artin–Schelter Gorenstein algebras are exhibited. Moreover, interesting interactions between the classification of Artin–Schelter regular algebras and that of quasi-Fano algebras as well as the classification of Artin–Schelter Koszul algebras and that of Frobenius Koszul algebras (via Koszul duality) are explained. Applications of algebraic geometry techniques to classification problems of Fano algebras and graded Frobenius algebras are also presented.

In the research article by Nakanishi two kinds of periodicities of mutations of general cluster algebras and their connections with systems of algebraic relations called T-systems and Y-systems as well as dilogarithm identities are discussed. The first one is the periodicity of exchange matrices (or quivers) under a sequence of mutations. The second one is the (stronger) periodicity of seeds under a sequence of mutations. For any sequence of mutations under which exchange matrices are periodic, Nakanishi defines the associated T-systems and Y-systems, which for regular sequences of mutations coincide with the known classical T-systems and Y-systems. Furthermore, for any sequence of mutations under which seeds are periodic, the associated dilogarithm identity is formulated. The author proves these dilogarithm identities when the exchange matrices are skew symmetric.

The article by de la Peña and Skowroński surveys old and new results on the properties of the Tits quadratic forms of tame finite dimensional algebras over an algebraically closed field. One of the main aims of the article is to outline the crucial ingredients of the proof of the recent result by Brüstle, de la Peña and Skowroński asserting that the tameness of a strongly simply connected algebra is equivalent to the weak nonnegativity of the associated Tits quadratic form. In the article several important applications of this result are exhibited. Furthermore, criteria for a strongly simply connected algebra to be representation-finite, of finite growth, of polynomial growth are presented. The authors present also several results on the values of the Tits forms, as well as the related Euler forms, on the dimension vectors of finite dimensional indecomposable modules over tame strongly simply connected algebras. Finally, the realization of positive roots of the Tits forms of tame algebras as the dimension vectors of indecomposable modules is discussed.

The main aim of the research article by Ringel is to give a complete classification of the minimal representation-infinite special biserial algebras over an algebraically closed field and to describe the structure of module categories of these algebras. In particular, Ringel proves that the minimal representation-infinite special biserial algebras without nodes are cycle algebras, the barbell algebras with nonserial bars and the wind wheel algebras. Furthermore, it is shown in the article that a minimal representation-infinite algebra is special biserial if and only if its universal Galois covering is interval-finite, with free Galois group, and any its finite convex subcategory is representation-finite. An interesting new phenomena discovered by Ringel is that some minimal representation-infinite special biserial algebras (namely the barbell algebras) are not of polynomial growth.

In the article by Simson a current overview on the representation theory of coalgebras over a field is presented. Simson discusses the concepts of tame comodule type, of discrete comodule type, of polynomial growth, and of wild comodule type for a wide class of coalgebras intensively investigated during the last decade. In particular, the author shows that the tame-wild dichotomy holds for a wide class of coalgebras of infinite dimension over an algebraically closed field, including the semiperfect coalgebras and the incidence coalgebras of intervally finite partially ordered sets. Furthermore, basic tools and techniques applied in the study of coalgebras and their comodule categories

are presented. Characterizations of large classes of coalgebras of tame comodule type are also given.

The article by Zwara surveys old and recent results on singularities of Zariski closures of orbits in module varieties of finite dimensional algebras (and representations of quivers) over an algebraically closed field under the actions of general linear groups via conjugations. One of the main aims of the article is to discuss the types of singularities which may occur in the orbit closures of module varieties and links with classical types of singularities occurring in algebraic geometry (Schubert varieties of flag varieties). Zwara presents in the article several tools (degenerations, transversal slices, desingularizations, hom-controlled exact functors) allowing to study the singularities of orbit closures of module varieties. The author's results on singularities for degenerations of modules of codimension at most two are presented in details. Furthermore, the equations of orbit closures and generic singularities are discussed. The article contains many examples and open problems, motivating further study of orbit closures of modules and their singularities.

It is our hope that the wide scope of the collection of articles in the book will give a panoramic view of some recent trends in the representation theory of algebras and its exciting interaction with cluster algebras and categories, representation theory of finite groups, commutative and noncommutative geometry, commutative algebra, homological algebra, quantum algebras, algebraic combinatorics, theoretical physics, topology, and representation theory of coalgebras. This interaction was responsible for much of enormous progress we have seen during the last three decades in representation theory of algebras. The articles are self-contained and addressed to researchers and graduate students in algebra as well as a broader mathematical community. The large number of open problems posed in the articles gives also a perspective of further research.

We express our gratitude to all authors contributing in this book and the referees for their assistance. Particular thanks are due to Jerzy Białkowski for his computer help in proper edition of the articles. We also thank the European Mathematical Society Publishing House for publication of this collection of articles and Manfred Karbe and Irene Zimmermann for very kind cooperation.

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Editors