Foreword

In a broad sense, the subject of Teichmüller theory is the study of moduli spaces for geometric structures on surfaces. This subject makes important connections between several areas in mathematics, including low-dimensional topology, hyperbolic geometry, dynamical systems theory, differential geometry, algebraic topology, representations of discrete groups in Lie groups, symplectic geometry, topological quantum field theory, string theory, and there are others.

The central object in this theory is the Teichmüller space of a surface. As is well-known, this space can be seen from different points of view. It is a space of equivalence classes of conformal structures on the surface, a space of equivalence classes of hyperbolic metrics on this surface, and a space of equivalence classes of representations of the fundamental group of the surface into a Lie group (primarily, $PSL(2, \mathbb{R})$, but there are several others). These three points of view are equally important in the study of Teichmüller space and, in several situations, they cannot be clearly separated. Furthermore, Teichmüller space inherits from these different points of view various structures, including several interesting metrics (Teichmüller, Weil–Petersson, Thurston, Bergman, Carathéodory, Kähler–Einstein, McMullen, etc.), a natural complex structure, a symplectic structure, a real analytic structure, the structure of an algebraic set, cellular structures, various boundary structures, a natural discrete action by the mapping class group, a quantization theory of its Poisson and symplectic structures, a measure-preserving geodesic flow, a horocyclic flow, and the list of structures goes on and on. Teichmüller theory is growing at a fantastic rate, and the richness of this theory is to a large extent a consequence of the diversity and the richness of the structures that Teichmüller space itself carries.

The purpose of this Handbook is to give a comprehensive picture of the classical and of the recent developments in Teichmüller theory. The range of this picture will hopefully include all the aspects mentioned above although, because new ideas and new connections come out regularly in this theory, the picture will necessarily be incomplete. Nevertheless we hope that the Handbook will reflect the beauty, the liveliness and the richness of Teichmüller theory.

I have tried to divide this first volume of the Handbook into parts. There were several possibilities, none of which imposed itself as being more natural or more efficient than the others. I finally ended up with the following division into four parts.[1]

- The metric and the analytic theory, 1
- The group theory, 1
- Surfaces with singularities and discrete Riemann surfaces
- The quantum theory, 1

[1] The titles of some parts are followed by the numeral 1 because they will be continued in Volume II of the Handbook.

Volume II of the Handbook will contain more material on the analytic and the metric theory, on the group theory, on the quantum theory, as well as sections on cluster algebras, on representation theory and higher Teichmüller theory, on complex projective structures, on the Teichmüller geodesic flow and other dynamical aspects, and on the Grothendieck–Teichmüller theory.

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