Preface

Roughly twenty years after their appearance on the mathematical scene, quantum groups have played a significant role in various branches of mathematics. This volume contains one lecture course and three research expositions devoted to various aspects of this subject. We briefly introduce these contributions and relate them to some of the main themes of the theory.

The first contribution, "Lectures on tensor categories" provides an introduction to tensor categories. It is based on the lecture notes (by D. Calaque) of a course taught by P. Etingof in Strasbourg in June 2003. While the origins of the subject of tensor categories lie in the Tannaka–Krein duality for compact groups, it was after the construction by Reshetikhin and Turaev of invariants of 3-manifolds using a tensor category obtained from the quantum group $U_q(\mathfrak{sl}_2)$ for q a root of unity, and the interpretation by Drinfeld of Kohno's computation of the monodromy of the Knizhnik–Zamolodchikov differential system in terms of quasi-Hopf algebras that its relations with the circle of problems called "the theory of quantum groups" were firmly established. Some of the main subsequent results in this theory – notably the Kazhdan–Lusztig theorem identifying tensor categories arising from rational conformal field theories and from quantum groups, and the solution by Etingof and Kazhdan of the quantization problem of Lie bialgebras – were based on these relations. The lectures require only minimal prerequisites in category theory and in Hopf algebras, and lead the reader to topics of recent research, like the program of classification of tensor categories.

The second contribution, "The Drinfeld associator of gl(1|1)", by J. Lieberum, deals with associators and Vassiliev invariants of knots. The subject of associators started in 1989–90 with their introduction by Drinfeld as universal objects for the construction of quasitriangular quasi-Hopf algebras; it was related to Vassiliev invariants by the Kontsevich integral. While the set of associators $(\lambda, \Phi) \in \mathbb{C} \times \exp(\mathfrak{f}_2)$ is "large" (in bijection with the prounipotent version of the Grothendieck–Teichmüller group; here $\mathfrak{f}_2$ is the topologically free complex Lie algebra with two generators), its image under "standard" specialization maps is usually small; e.g., it follows from Drinfeld's theorems on quasi-Hopf algebras that the images of associators (with $\lambda \neq 0$ fixed) in $U(\mathfrak{g})^{\otimes 3}[[\hbar]]$ (where $\mathfrak{g}$ is a simple Lie algebra) are all related by twists by elements of $(U(\mathfrak{g})^{\otimes 2})^{\mathfrak{g}}[[\hbar]]^{\times}$. In his contribution (a part of his Habilitationsschrift), Lieberum studies the image of associators in $U(\mathfrak{g})^{\otimes 3}[[\hbar]]$, where $\mathfrak{g}$ is the Lie superalgebra gl(1|1). It is shown that the images of the even associators (again with $\lambda \neq 0$ fixed; even means $\Phi(A, B) = \Phi(-A, -B)$) are *all the same*. For this, the specialization map to gl(1|1) is shown to factor through certain quotients of the algebras of trivalent diagrams, in which the uniqueness of even associators is proved. This result is supplemented with an explicit formula for this even associator. In the second part of the paper, the author relates the associator of gl(1|1) with Viro's generalization of the multivariable Alexander polynomial.

The third contribution is “Integrable systems associated with elliptic algebras”, by Odesskii and Rubtsov. The elliptic algebras here are generalizations, developed by Feigin–Odesskii, of Sklyanin’s elliptic algebras. The main purpose of this contribution is the construction of new integrable systems in elliptic algebras. The authors use a general method of construction, which was rediscovered several times in various contexts, and is related to separation of variables; its main input is a family of elements in an algebra satisfying certain commutation condition; the commuting hamiltonians are then ratios of noncommutative (“Cartier–Foata”) determinants. While in previous applications of this method the commutation relations were ensured by the fact that the elements belonged to different factors of the tensor power of an algebra (whence the relation to separation of variables), the reasons why the commutation relations are satisfied here are different and are related to the structure of elliptic algebras. The resulting integrable systems generalize the antiperiodic solid-on-solid (SOS) models, which are based on Felder’s elliptic quantum group $E_{\tau,\eta}(\mathrm{sl}_2)$.

The fourth contribution “On the automorphisms of $U_q^+(\mathfrak{g})$”, by N. Andruskiewitsch and F. Dumas, deals with a problem of pure algebra: to compute the group of algebra automorphisms of the “nilpotent part” $U_q^+(\mathfrak{g})$ of a quantum enveloping algebra $U_q(\mathfrak{g})$. While a complete solution had been obtained by Alev and Dumas for $\mathfrak{g}$ of type A_2, the authors obtain significant partial results in the case where $\mathfrak{g}$ is of type B_2.

Two of these contributions (the second and the third) were presented during the 72th RCP meeting “Quantum Groups” in Strasbourg (June 2003). I wish to end this preface by thanking Vladimir Turaev for the very pleasant atmosphere during our joint organization of the meeting in Strasbourg, and all the referees for their invaluable help with this volume.

Strasbourg, May 2008 *Benjamin Enriquez*