

Preface

The Twelfth International Conference on Representations of Algebras and Workshop (ICRA XII) was held at the Faculty of Mathematics and Computer Science, Nicolaus Copernicus University in Toruń, Poland, from 15 to 24 August, 2007. The ICRA XII was attended by the remarkable large number of 250 researchers and graduate students from 30 countries of all parts of the world. We thank for their lively interest and for providing a very pleasant audience.

The Scientific Advisory ICRA XII Committee consisted of R. Bautista (Morelia, Mexico), R.-O. Buchweitz (Toronto, Canada), M. C. R. Butler (Liverpool, United Kingdom), W. Crawley-Boevey (Leeds, United Kingdom), V. Dlab (Ottawa, Canada), Y. A. Drozd (Kiev, Ukraine), K. Erdmann (Oxford, United Kingdom), D. Happel (Chemnitz, Germany), B. Huisgen-Zimmermann (Santa Barbara, United States), B. Keller (Paris, France), H. Lenzing (Paderborn, Germany), M.-P. Malliavin (Paris, France), H. Merklen (Sao Paulo, Brazil), J. A. de la Peña (Mexico City, Mexico), M. I. Platzeck (Bahia Blanca, Argentina), I. Reiten (Trondheim, Norway), C. M. Ringel (Bielefeld, Germany), A. V. Roiter (Kiev, Ukraine), D. Simson (Toruń, Poland), A. Skowroński (Chairman, Toruń, Poland), S. O. Smalø (Trondheim, Norway), K. Yamagata (Tokyo, Japan), Y. Zhang (Beijing, China).

The Local Organizing ICRA XII Committee was formed by A. Skowroński (Chairman), J. Białkowski, G. Bobiński, R. Bocian, P. Dowbor, S. Kasjan, J. Kosakowska, Z. Leszczyński, P. Malicki, Z. Pogorzały, D. Simson, G. Zwara.

We would like to thank the members of the Committees as well the leaders of research groups for the advices, help and cooperation making the ICRA XII very successful. We are also grateful to the Faculty of Mathematics and Computer Science of the Nicolaus Copernicus University and the Polish Research Council for a substantial financial support.

According to a tradition in the area, the ICRA XII was divided into two parts: the Workshop and the Conference. The ICRA XII Workshop consisted of seven mini-courses of three hours each given by Susumu Aiki (Kyoto), Ragnar-Olaf Buchweitz (Toronto), Osamu Iyama (Nagoya), Bernhard Keller (Paris), Steffen Koenig (Köln), Jose Antonio de la Peña (Mexico City), and Markus Reineke (Wuppertal). The ICRA XII Conference comprised 144 talks (24 plenary talks, 120 talks in parallel sessions), among them fourteen one hour plenary lectures given by Igor Burban (Mainz), William Crawley-Boevey (Leeds), Harm Derksen (Ann Arbor), Yuriy Drozd (Kiev), Karin Erdmann (Oxford), Iain Gordon (Edinburgh), Bernard Leclerc (Caen), Helmut Lenzing (Paderborn), Steffen Oppermann (Köln), Idun Reiten (Trondheim), Christine Riedtmann (Bern), Claus Michael Ringel (Bielefeld), Jie Xiao (Beijing), and Andrei Zelevinsky (Boston).

A new event in more than 30 years of ICRA Conferences history took place in Toruń: the ICRA Award, established by the ICRA Scientific Advisory Committee during the

ICRA XI Conference held in Pátzcuaro, Mexico, in August 2004, for outstanding work by young mathematicians working in the area of representation theory of algebras. The ICRA AWARD 2007 was given to Osamu Iyama from the Nagoya University, Japan, for his original and influential work on developing a higher theory for almost split sequences and Auslander correspondence, and his subsequent work on Calabi–Yau categories, which have strong connections with the cluster algebras of Fomin–Zelevinsky.

This book contains fifteen expository survey articles on recent developments and trends in the area of representation theory of algebras and related topics, reflecting the topics of some lectures presented during the ICRA XII Workshop and Conference held in Toruń.

We now briefly describe the contents of the articles.

The article by Ariki surveys development on the modular representation theory of finite dimensional Hecke algebras in the last decade and explains current views on the topic. The author reviews basics on the Kazhdan–Lusztig canonical bases, the canonical basic sets by Geck and Jacon, Kashiwara’s crystal theory, and the Fock space theory, developed by him and his collaborators. Moreover, several applications of the theory of crystals and canonical bases, including the modular branching rule and the representation type of Hecke algebras, are presented. In the final part of the article, the author gives a new and very reader friendly explanation of Rouquier’s theory of quasihereditary covers of cyclotomic Hecke algebras in terms of the category \mathcal{O} for the rational Cherednik algebra.

Bobirski, Riedtmann and Skowroński present old and new results concerning the structure of the algebras of semi-invariant polynomial functions on the affine varieties of linear representations of quivers of a given dimension vector with respect to conjugate actions of products of general linear groups as well as the geometry of the sets of common zeros of generating nonconstant semi-invariants. In particular, the result by Derksen and Weyman describing spanning sets of the underlying vector spaces of the algebras of semi-invariants in terms of determinantal semi-invariants of quivers defined by Schofield is presented. The main geometric question discussed in the article is when the algebra of semi-invariants of a quiver (or the associated zero set) is a complete intersection. The theorem by Skowroński and Weyman characterizing the Dynkin and Euclidean quivers as those connected quivers for which all algebras of semi-invariants are complete intersections, as well as the results by Riedtmann and Zwara concerning the geometry of zero sets of semi-invariants of Dynkin and Euclidean quivers are described.

The article by Burban and Drozd reviews results on properties of maximal Cohen–Macaulay modules over surface singularities. In particular, old and new results on the representation type (finite, tame, wild) of the category of maximal Cohen–Macaulay modules over surface singularities are presented. Basic tools of the study of Cohen–Macaulay modules over surface singularities, including the Macaulayfication functor, the algebraic and geometric McKay correspondence for simple surface singularities, Kahn’s reduction of the classification of Cohen–Macaulay modules over minimally elliptic singularities to the classification of vector bundles on projective curves of arithmetic genus one, are explained in detail. As an application, it is concluded that

the simple elliptic and cusp singularities, as well their quotients by a finite automorphism group, have tame Cohen–Macaulay representation type (results by Kahn, Drozd, Greuel, Kashuba).

Carlson gives a survey of the theory of rank varieties for modules over finite groups, finite group schemes and other algebras. The rank varieties are frequently homeomorphic to the cohomological support varieties defined in terms of the action of the cohomology ring of the algebra on the cohomology ring of the module. In this theory a prominent role is played by the results asserting the finite generation of the cohomology rings established by Evens and Venkov for finite groups and Friedlander and Suslin for arbitrary group schemes. The article is intended to exhibit the progression of the ideas over the three decades and recent generalization of rank varieties to the theory of π -points of group schemes by Friedlander and Pevtsova. Moreover, an emerging representation theory of modules of constant Jordan type developed recently by Carlson, Friedlander, Pevtsova and Suslin is presented.

The main aim of the article by Erdmann and Skowroński is to present results on finite dimensional algebras over an algebraically closed field which are periodic when considered as modules over their enveloping algebras. These algebras are selfinjective and have periodic module categories with respect to the action of the syzygy operators. In the article, the known classes of periodic algebras and their roots in the theory of finite groups, algebraic topology and commutative algebra are exhibited. In particular, the Morita equivalence classes of all periodic blocks of group algebras of finite groups are described completely. Moreover, a new class of periodic algebras called deformed mesh algebras of generalized Dynkin type, containing the class of preprojective algebras of generalized Dynkin type, is described. These algebras, by a recent result by Białkowski, Erdmann and Skowroński, are exactly the algebras for which the third syzygy operator permutes the isomorphism classes of simple modules.

In the contribution of Geiss, Leclerc and Schröer, the representation theory of preprojective algebras of Dynkin type is applied to give a representation theoretic treatment of the class of cluster algebras discovered by Bernstein, Fomin and Zelevinsky in their work on total positivity and the geometry of double Bruhat cells in semisimple algebraic groups. Results of a series of the authors' recent papers are presented and illustrated by examples. In particular, it is shown that the cluster monomials of these cluster algebras belong to the dual of Lusztig's semicanonical basis, and hence are linearly independent. In the article, new cluster algebra structures on the coordinate rings of partial flag varieties are defined and discussed. In this connection, interesting Frobenius subcategories of the module categories of preprojective algebras of Dynkin type are investigated.

The article by Gordon is a discursive introduction to the representation theory of symplectic reflection algebras, exhibited by Etingof and Ginzburg a few years ago. In the article, the symplectic reflection algebras are defined as graded deformations of the smash product algebras given by actions of symplectic reflection groups on the associated algebras of regular functions on the finite dimensional complex space. In the two-dimensional case, the symplectic reflection algebras are Morita equivalent to the deformed preprojective algebras of affine Dynkin quivers discovered by

Crawley-Boevey and Holland. Gordon discusses recent results on the representation theory of two main examples of symplectic reflection algebras: rational Cherednik algebras and deformations of tensor products on deformed preprojective algebras of affine Dynkin quivers. In the case of symplectic reflection algebras associated to the symmetric group their representation theory is related to sheaves on the Hilbert scheme of points on the plane.

In the article by Iyama, the author's higher theory of almost split sequences and Auslander correspondence for finite dimensional algebras over fields and orders over complete local rings is introduced. In particular, the basic properties of n -Auslander algebras, given by the homological conditions “global dimension at most $n + 1$ ” and “dominant dimension at least $n + 1$ ”, are described. A prominent role in this investigation is played by n -cluster tilting subcategories, giving representation theoretic realizations of n -Auslander algebras. Moreover, important examples and constructions of n -cluster tilting subcategories are exhibited and discussed.

Jørgensen considers in his article recent results on links between the theory of Calabi–Yau categories and rational homotopy theory, via the differential graded algebra structures of the singular cochain complexes of topological spaces with coefficients in fields of characteristic zero. One of the highlights is the author's theorem asserting that a simply connected topological space has an n -dimensional Poincaré duality if and only if the compact derived category of the associated singular cochain complex differential graded algebra is an n -Calabi–Yau category. The main part of the article is devoted to applications of the Auslander–Reiten theory to the compact derived categories of Gorenstein differential algebras, including a description of results from the doctoral thesis of Karsten Schmidt. In particular, the shape of all Auslander–Reiten components of Gorenstein differential graded algebras is described, and the finiteness of the number of these components is shown to be equivalent to the dimension of the cohomology algebra being bounded by two.

The article by Kasjan surveys results on the applications of model theoretical tools (ultraproduct techniques, compactness theorem, quantifier elimination) to geometric and homological problems concerning finite dimensional algebras. The starting point of the discussion are the theorem on finite axiomatizability of the class of algebras of finite representation type by Herrmann, Jensen and Lenzing, and Gabriel's theorem asserting that the finite representation type is open. Recent results of the author concerning the open question whether the algebras of finite representation type induce open \mathbb{Z} -schemes are presented. A second geometric question discussed is the following: is the tame representation type open, that is, the class of algebras of tame representation type induces an open subset of the variety of algebras of any fixed dimension (more generally, induces an open \mathbb{Z} -scheme)? It is shown that these questions are strongly related to finite axiomatizability of the class of algebras of tame representation type. As an application, it is concluded that the class of strongly simply connected algebras of tame representation type induces an open \mathbb{Z} -scheme (hence is open) in every dimension.

Keller discusses in his contribution the definition of a Calabi–Yau category introduced by Kontsevitch about ten years ago. He reviews basic notions on triangulated

categories related to the Calabi–Yau property and describes two important classes of examples: Calabi–Yau categories arising as orbit categories and Calabi–Yau categories arising as subcategories of derived categories. In particular, 2-Calabi–Yau categories given by preprojective algebras and 3-Calabi–Yau categories with potentials are exhibited.

The aim of the article by Koenig is to provide an elementary introduction to the structure and representation theory of diagram algebras, including Brauer algebras, Temperley–Lieb algebras, partition algebras, Hecke algebras, affine Temperley–Lieb algebras, affine Hecke algebras, Birman–Murakami–Wenzl algebras, walled Brauer algebras, party algebras, occurring naturally in group theory, invariant theory, combinatorics, statistical mechanics, knot theory, algebraic Lie theory and number theory. The author reviews recent development focussing on common structure results of various types of diagram algebras. A prominent role is played by the concept of cellular algebras introduced by Graham and Lehrer. In the article, homological properties of diagram algebras are discussed in detail.

The article by Lenzing and de la Peña reviews old and new results on the impact given by the spectrum of Coxeter transformation of algebras of finite global dimension on the structure and representation theory of algebras. The class of algebras under discussion in the article includes the hereditary algebras, canonical algebras, extended canonical algebras and supercanonical algebras, for which the spectrum of the Coxeter transformation contains enough information to recover the algebras up to derived equivalence. In the article, exciting links of the representation theory of finite dimensional algebras to the singularity theory via the derived categories of graded singularities associated to weighted projective lines, invoking Orlov’s theorem, are exhibited. Moreover, properties of the stable categories of vector bundles on weighted projective lines are described.

The article by Reineke provides an introduction to the moduli spaces of representations of quivers, offering the geometric approach to their classification. The author reviews basics from the Geometric Invariants Theory and discusses stability and semistability of representations of quivers. One of the main problems considered in the article towards a study of the global geometry of moduli spaces of representations of quivers is the existence and structure of their cell decompositions. In this direction, the author’s results concerning the Betti numbers of quiver moduli spaces as well as counting rational points over finite fields are presented. The role of Hall algebras in calculations of these numbers is also explained. Moreover, recent results by the author and Johannes Engel concerning the smooth models of quiver moduli spaces and Hilbert schemes are presented.

Finally, the contribution of Skowroński and Yamagata surveys old and new results on the representation theory of selfinjective algebras of quasitilted type, that is, the finite dimensional algebras over fields having Galois coverings by the repetitive algebras of quasitilted algebras (endomorphism algebras of tilting objects in hereditary abelian categories). Basic results from the Auslander–Reiten theory, representation theory of quasitilted algebras and Galois coverings are reviewed. A prominent role is played by

the theory of positive Galois coverings of selfinjective algebras developed by the authors during the last fifteen years. In particular, it is explained that the selfinjective algebras whose Auslander–Reiten quiver admits an acyclic generalized standard component (respectively, all Auslander–Reiten components are generalized standard) are socle deformations of selfinjective algebras of quasitilted type. Invariance of the class of selfinjective algebras of quasitilted type under equivalences of stable module categories is also discussed.

It is our hope that the wide scope of the collection of articles in the book will give a panoramic view of some recent trends in the representation theory of algebras and its exciting interaction with algebraic and symplectic geometry, commutative algebra, homological algebra, quantum groups, theoretical physics, algebraic combinatorics, topology and model theory. This interaction was responsible for much of the enormous progress we have seen during the last three decades in representation theory of algebras. The articles are self-contained and addressed to researchers and graduate students in algebra as well as a broader mathematical community. The large number of open problems posed in the articles gives also a perspective for further research.

I express my gratitude to all authors contributing in this book and the referees for their assistance. Particular thanks are due to Jerzy Białkowski for his computer help in proper edition of the articles. I also thank the European Mathematical Society Publishing House for publication of this collection of articles and Manfred Karbe and Irene Zimmermann for very kind cooperation.

Toruń, August 2008

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