Contents

1 Examples of algebra-geometry correspondences 1

1.1 Locally compact spaces and commutative C^*-algebras 1
1.2 Vector bundles, finite projective modules, and idempotents 15
1.3 Affine varieties and finitely generated commutative reduced algebras . 21
1.4 Affine schemes and commutative rings 24
1.5 Compact Riemann surfaces and algebraic function fields 25
1.6 Sets and Boolean algebras 26
1.7 From groups to Hopf algebras and quantum groups 27

2 Noncommutative quotients 43

2.1 Groupoids 43
2.2 Groupoid algebras 48
2.3 Morita equivalence 60
2.4 Morita equivalence for C^*-algebras 69
2.5 Noncommutative quotients 75
2.6 Sources of noncommutative spaces 82

3 Cyclic Cohomology 83

3.1 Hochschild cohomology 85
3.2 Hochschild cohomology as a derived functor 91
3.3 Deformation theory 98
3.4 Topological algebras 108
3.5 Examples: Hochschild (co)homology 111
3.6 Cyclic cohomology 120
3.7 Connes' long exact sequence 132
3.8 Connes' spectral sequence 136
3.9 Cyclic modules 139
3.10 Examples: cyclic cohomology 144

4 Connes–Chern character 150

4.1 Connes–Chern character in K-theory 150
4.2 Connes–Chern character in K-homology 163
4.3 Algebras stable under holomorphic functional calculus 180
4.4 A final word: basic noncommutative geometry in a nutshell 184

Appendices 186

A Gelfand–Naimark theorems 186
A.1 Gelfand's theory of commutative Banach algebras 186
A.2 States and the GNS construction 190

B Compact operators, Fredholm operators, and abstract index theory 197

C Projective modules 204

D Equivalence of categories 206

Bibliography 209

Index 219