

Preface

This book is about metric spaces of nonpositive curvature in the sense of Busemann, that is, metric spaces whose distance function is convex. We have also included a systematic introduction to the theory of geodesics and related matters in metric spaces, as well as a detailed presentation of a few facets of convexity theory that are useful in the study of nonpositive curvature.

We have tried to start from first principles and to give full proofs, but there are two exceptions, which occur several times in examples that are spread through the book. These exceptions are the following:

- we give without proof some classical well-known facts about hyperbolic spaces \mathbb{H}^n ;
- we refer to some elements of the theory of Teichmüller space.

The material that we present in connection with these spaces is used as an illustration for the general concepts that are developed. It can be asserted without exaggeration that Teichmüller space is the most beautiful space ever, and the metric theory of that space, in particular its convexity properties, is most interesting.

This book can be divided into three parts. Part I (Chapters 1 to 4) contains basic material on metric spaces, Part II (Chapters 5 to 7) concerns convexity in vector spaces and Part III (Chapters 8 to 12) concerns convexity in metric spaces and related matters.

Each chapter contains an introduction in which we describe its content. At the end of each chapter we have included a few notes, some of them historical and others indicating some further developments. In both cases, there is no pretention for completeness. The only reason for which a note is included is that the author of this book thinks that it is interesting information.

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