

Abstract

We introduce a variant of stable logarithmic maps, which we call *punctured logarithmic maps*. They allow an extension of logarithmic Gromov–Witten theory in which marked points have a negative order of tangency with boundary divisors.

As a main application we develop a gluing formalism which reconstructs stable logarithmic maps and their virtual cycles without expansions of the target, with tropical geometry providing the underlying combinatorics.

Punctured Gromov–Witten invariants also play a pivotal role in the intrinsic construction of mirror partners by the last two authors, conjecturally relating to symplectic cohomology, and in the logarithmic gauged linear sigma model in work of Qile Chen, Felix Janda and Yongbin Ruan.

Keywords. logarithmic Gromov–Witten invariant, punctured Gromov–Witten invariant, punctured map, punctured curve, puncturing, functorial tropicalization, tropical punctured map, tropical moduli, tropical type, marking by tropical type, basic monoid, basic logarithmic structure, negative contact order, gluing formula, splitting, Artin fan, perfect obstruction theory, virtual fundamental class, logarithmic geometry, logarithmic stack

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