

Contents

1	Introduction	1
1.1	Scope and motivation	1
1.2	Main features of punctured Gromov–Witten theory	3
1.3	Statements of main results	4
1.4	Organization of the paper	7
1.5	Relation to other work	8
1.6	Conventions	9
2	Punctured maps	11
2.1	Definitions	11
2.2	The tropical interpretation	21
2.3	Basicness	31
2.4	Global contact orders and global types	36
2.5	Puncturing log-ideals	43
2.6	Targets with monodromy	53
3	The stack of punctured maps	61
3.1	Stacks of punctured curves	61
3.2	Stacks of punctured maps marked by tropical types	66
3.3	Boundedness	71
3.4	Valuative criterion	74
3.5	Idealized smoothness of $\mathfrak{M}(\mathcal{X}/B, \tau) \rightarrow B$	77
4	The perfect obstruction theory	89
4.1	Obstruction theories for logarithmic maps from pairs	89
4.2	Obstruction theories for punctured maps with point conditions	97
4.3	Punctured Gromov–Witten invariants	100
5	Splitting and gluing	103
5.1	Splitting punctured maps	103
5.2	Gluing punctured maps to \mathcal{X}/B	106
5.3	Evaluation stacks and gluing at the virtual level	117
5.4	Gluing in the degeneration setup	128
A	Contact orders	133
A.1	Family of contact orders of Artin cones	134

A.2 Family of contact orders of Zariski Artin fans	136
A.3 Connection with the global contact orders of Section 2.4	138
B Charts for morphisms of log stacks	141
C Functorial tropicalization and the category of points	145
C.1 Tropicalization of fine log schemes	145
C.2 Tropicalization of fine log algebraic stacks	149
C.3 Tropicalization in the log smooth case	151
References	153