Abstract

We study the construction of the Φ_3^3-measure and complete the program on the (non-)construction of the focusing Gibbs measures, initiated by Lebowitz, Rose, and Speer [J. Statist. Phys. 50 (1988), no. 3-4, 657–687]. This problem turns out to be critical, exhibiting the following phase transition. In the weakly nonlinear regime, we prove normalizability of the Φ_3^3-measure and show that it is singular with respect to the massive Gaussian free field. Moreover, we show that there exists a shifted measure with respect to which the Φ_3^3-measure is absolutely continuous. In the strongly nonlinear regime, by further developing the machinery introduced by the authors, we establish non-normalizability of the Φ_3^3-measure. Due to the singularity of the Φ_3^3-measure with respect to the massive Gaussian free field, this non-normalizability part poses a particular challenge as compared to our previous works. In order to overcome this issue, we first construct a σ-finite version of the Φ_3^3-measure and show that this measure is not normalizable. Furthermore, we prove that the truncated Φ_3^3-measures have no weak limit in a natural space, even up to a subsequence.

We also study the dynamical problem for the canonical stochastic quantization of the Φ_3^3-measure, namely, the three-dimensional stochastic damped nonlinear wave equation with a quadratic nonlinearity forced by an additive space-time white noise (= the hyperbolic Φ_3^3-model). By adapting the paracontrolled approach, in particular from the works by Gubinelli, Koch, and the first author [J. Eur. Math. Soc. 26 (2024), no. 3, 817–874] and by the authors [Mem. Amer. Math. Soc. 304 (2024), no. 1529], we prove almost sure global well-posedness of the hyperbolic Φ_3^3-model and invariance of the Gibbs measure in the weakly nonlinear regime. In the globalization part, we introduce a new, conceptually simple and straightforward approach, where we directly work with the (truncated) Gibbs measure, using the Boué–Dupuis variational formula and ideas from theory of optimal transport.

Keywords. Φ_3^3-measure, stochastic quantization, stochastic nonlinear wave equation, nonlinear wave equation, Gibbs measure, paracontrolled calculus

Mathematics Subject Classification (2020). Primary 60H15; Secondary 81T08, 60L40, 35L71, 35K15

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