Abstract

The Seiberg–Witten Floer spectrum is a stable homotopy refinement of the monopole Floer homology of Kronheimer and Mrowka. The Seiberg–Witten Floer spectrum was defined by Manolescu for closed, spin^c 3-manifolds with $b_1 = 0$ in an S^1-equivariant stable homotopy category and has been producing interesting topological applications. Lidman and Manolescu showed that the S^1-equivariant homology of the spectrum is isomorphic to the monopole Floer homology.

For closed spin^c 3-manifolds Y with $b_1(Y) > 0$, there are analytic and homotopy-theoretic difficulties in defining the Seiberg–Witten Floer spectrum. In this memoir, we address the difficulties and construct the Seiberg–Witten Floer spectrum for Y, provided that the first Chern class of the spin^c structure is torsion and that the triple-cup product on $H^1(Y;\mathbb{Z})$ vanishes. We conjecture that its S^1-equivariant homology is isomorphic to the monopole Floer homology.

For a 4-dimensional spin^c cobordism X between Y_0 and Y_1, we define the Bauer–Furuta map on these new spectra of Y_0 and Y_1, which is conjecturally a refinement of the relative Seiberg–Witten invariant of X. As an application, for a compact spin 4-manifold X with boundary Y, we prove a $\frac{10}{8}$-type inequality for X which is written in terms of the intersection form of X and an invariant $\kappa(Y)$ of Y.

In addition, we compute the Seiberg–Witten Floer spectrum for some 3-manifolds.

Keywords. Seiberg–Witten equations, Floer theory, stable homotopy, Conley index, spectral sections

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