

Abstract

We introduce a two parameter family of Dirac operators on quantum $SU(2)$ and analyse their properties from the point of view of non-commutative metric geometry. It is shown that these Dirac operators give rise to compact quantum metric structures, and that the corresponding two parameter family of compact quantum metric spaces varies continuously in Rieffel's quantum Gromov–Hausdorff distance. This continuity result includes the classical case where we recover the round 3-sphere up to a global scaling factor on the metric. Our main technical tool is a quantum $SU(2)$ analogue of the Berezin transform, together with its associated fuzzy approximations, the analysis of which also leads to a systematic way of approximating Lipschitz operators by means of polynomial expressions in the generators.

Keywords. quantum $SU(2)$, quantum metric spaces, Gromov–Hausdorff distance

Mathematics Subject Classification (2020). Primary 58B32; Secondary 58B34, 46L89, 46L30, 81R60

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