

Contents

1	Introduction	1
1.1	Notation and standing assumptions	6
1.2	Note added in proof	6
2	Compact quantum metric spaces	9
2.1	Definitions and basic properties	9
2.2	Quantum Gromov–Hausdorff distance	13
2.3	Finitely generated projective modules	18
3	Preliminaries on quantum $SU(2)$	23
3.1	The quantum enveloping algebra	24
3.2	Twisted derivations	25
3.3	Corepresentation theory	26
3.4	The Haar state	27
3.5	Circle actions	28
3.6	Analytic elements	29
3.7	The continuous field	32
4	Spectral geometry on quantum $SU(2)$	35
4.1	The horizontal and vertical Dirac operators	35
4.2	The origin of the Dirac operators	36
4.3	Bounded twisted commutators	39
4.4	Comparison with the classical Dirac operator	43
4.5	The real structure	45
4.6	The equivariance condition	49
4.7	Conjugating the Dirac element with the fundamental unitary	55
5	Quantum metrics on quantum $SU(2)$	61
5.1	The Podleś sphere revisited	61
5.2	Spectral projections and twisted derivations	65
5.3	Spectral bands as compact quantum metric spaces	68
5.4	Schur multipliers	72
5.5	Projecting onto the spectral bands	76
5.6	Quantum $SU(2)$ as a compact quantum metric space	78
6	The quantum Berezin transform	81
6.1	Definition of the Berezin transform	81
6.2	The image of the Berezin transform	84
6.3	Estimates on the Berezin transform	91

6.4	Approximation in the quantum Gromov–Hausdorff distance	99
7	Continuity results	103
7.1	Continuity of the fuzzy approximations	103
7.2	Uniformity of the fuzzy approximation	105
7.3	Continuity of quantum $SU(2)$	112
	References	115