

Contents

Preface	vii
1 Introduction	1
1.1 Setup	2
1.2 Minimizers	3
1.3 Epsilon regularity	4
1.4 Global generalized minimizers and classification	7
1.5 Structural results	10
1.6 Critical points	12
1.7 Plan of the notes	14
2 Density bounds, compactness, variations, and monotonicity	17
2.1 Preliminaries	17
2.2 Blow-up of minimizers	21
2.3 Compactness for generalized minimizers	25
2.4 Elementary global generalized minimizers	26
2.5 Variational identities	29
2.6 Monotonicity formulae	38
3 Pure jumps and triple junctions	49
3.1 Epsilon-regularity statements	50
3.2 Regularity at pure jumps: Preliminaries	53
3.3 Lipschitz approximation	55
3.4 Regularity at pure jumps: Decay lemmas	63
3.5 Regularity at pure jumps: Conclusion	70
3.6 Triple junctions, closeness at all scales	72
3.7 Proof of Theorem 3.1.2	79
3.8 Proof of Theorem 1.3.3 for pure jumps and triple junctions	83
4 The Bonnet–David rigidity theorem for cracktips	87
4.1 Main statement and consequences	87
4.2 An overview of the ideas in the proof of Theorem 4.1.1	91
4.3 The absence of pockets	93
4.4 The David–Léger rigidity theorem and consequences	95
4.5 The harmonic conjugate	105
4.6 The level sets of the harmonic conjugate: Part I	106
4.7 The level sets of the harmonic conjugate: Part II	113

4.8	The level sets of the harmonic conjugate: Part III	119
4.9	A special bounded connected component of K	121
4.10	Proof of Theorem 4.1.1	127
5	Epsilon regularity at the cracktip	137
5.1	Rescaling and reparametrization	141
5.2	First linearization	149
5.3	Spectral analysis	156
5.4	The linear three annuli property	166
5.5	Second linearization and proof of Theorem 5.0.1	171
6	Some consequences of the epsilon-regularity theory	181
6.1	Main statements	181
6.2	Proof of Theorem 6.1.1	184
6.3	Proofs of Corollary 6.1.4 and Theorem 6.1.6	189
6.4	Reverse Hölder inequality for ∇u	193
6.5	Higher integrability of ∇u via the porosity of K	197
6.6	Proof of Corollary 6.1.7	206
A	Variational identities	209
B	Equivalence of SBV and classical formulations	215
B.1	Interior density lower bound	215
B.2	Boundary density lower bound	222
B.3	Density lower bounds for SBV minimizers	224
B.4	Equivalence of SBV and classical formulations	226
B.5	Proof of Corollary 2.1.4	228
B.6	Compactness of bounded minimizers through the SBV formulation	229
C	Useful results from elementary topology	233
D	Proof of Theorem 2.2.3	237
E	Hirsch's coarea inequality for Hölder maps	247
	References	249
	List of symbols	253
	Index	255