Abstract

We extend the theory of almost coherent modules that was introduced in *Almost ring theory* by Gabber and Ramero (2003). Then we globalize it by developing a new theory of almost coherent sheaves on schemes and on a class of "nice" formal schemes. We show that these sheaves satisfy many properties similar to usual coherent sheaves, i.e., the almost proper mapping theorem, the formal GAGA, etc. We also construct an almost version of the Grothendieck twisted image functor $f^!$ and verify its properties. Lastly, we study sheaves of p-adic nearby cycles on admissible formal models of rigid-analytic varieties and show that these sheaves provide examples of almost coherent sheaves. This gives a new proof of the finiteness result for étale cohomology of proper rigid-analytic varieties obtained before in Scholze's work *p-adic Hodge theory for rigid-analytic varieties* (2013).

Keywords. almost mathematics, rigid-analytic spaces, pro-etale cohomology

Mathematics Subject Classification (2020). Primary 13J07; Secondary 14F30, 14G22

Acknowledgments. I am very grateful to B. Bhatt, B. Conrad, S. Petrov, and D. B. Lim for many fruitful discussions. I express additional gratitude to B. Bhatt for bringing [10, Theorem 10.11] and [30] to my attention. I am thankful to B. Conrad for reading the first draft of this work and making useful suggestions on how to improve the exposition of this memoir. Part of this work was carried out at the mathematics department of the University of Michigan. I thank them for their hospitality. I heartfully thank O. Gabber, D. Hansen, B. Heuer, K. Kedlaya, P. Scholze, and K. Shimomoto for their valuable comments on the previous draft version of this work. The later stages of this work were done at the Max Planck Institute for Mathematics in Bonn and at the Institute For Advanced Study. I thank them for the excellent working conditions and funding.