

Contents

1	Introduction	1
2	The two Hodge bundles	13
2.1	The domain	13
2.2	The Hodge line bundle	14
2.3	The second Hodge bundle	16
2.4	I -trivialization of the second Hodge bundle	18
2.5	Accidental isomorphisms	19
3	Vector-valued modular forms	25
3.1	Representations of $O(n, \mathbb{C})$	25
3.2	Automorphic vector bundles	26
3.3	Tube domain realization	30
3.4	Fourier expansion	32
3.5	Geometry of Fourier expansion	34
3.6	Special orthogonal groups	38
3.7	Rankin–Cohen brackets	40
3.8	Higher Chow cycles on $K3$ surfaces	41
4	Witt operators	45
4.1	Ordinary pullback	45
4.2	Quasi-pullback	49
5	Canonical extension over 1-dimensional cusps	53
5.1	Siegel domain realization	53
5.2	Jacobi group	56
5.3	Partial toroidal compactification	58
5.4	Canonical extension	60
5.5	The Hodge line bundle at the boundary	62
5.6	Toroidal compactification	64
6	Geometry of Siegel operators	67
6.1	Invariant part for a unipotent group	68
6.2	The sub vector bundle \mathcal{E}_λ^J	72
6.3	The Siegel operator	77

7 Fourier–Jacobi expansion	81
7.1 Fourier–Jacobi and Fourier expansion	82
7.2 Geometric approach to Fourier–Jacobi expansion	83
7.3 Vector-valued Jacobi forms	86
7.4 Classical Jacobi forms	91
8 Filtrations associated to 1-dimensional cusps	99
8.1 J -filtration on \mathcal{E}	99
8.2 J -filtration on $\mathcal{E}_{\lambda,k}$	102
8.3 J -filtration and representations	106
8.4 Decomposition of Jacobi forms	109
9 Vanishing theorem I	113
9.1 Proof of Theorem 9.1	114
9.2 Vanishing of holomorphic tensors	115
10 Square integrability	119
10.1 Petersson metrics	119
10.2 Petersson metrics on the tube domain	122
10.3 Asymptotic estimates on the tube domain	124
10.4 Proof of Theorem 10.1	127
11 Vanishing theorem II	133
11.1 Lifting to the Lie group	134
11.2 Highest weight modules	136
11.3 Proof of Theorem 11.1	139
References	141
Index of Symbols	145