List of notations

Functional spaces

$\mathcal{B}$ Baouendi–Grisvard solution space of (1.25), used in Appendix A, *p. 21*

$\mathcal{H}^1_z$ Weighted H^1 space for boundary data with norm (1.22), *p. 20*

$\mathscr{H}_B$ Hilbert space of data triplets for the solvability of the linearized Burgers system, *p. 65*

$\mathscr{H}_{FP}$ Hilbert space with norm (3.5) of data triplets (f, δ_0, δ_1) for the kinetic toy model, *p. 50*

$\mathscr{H}_K$ Hilbert space with norm (2.14) of data triplets (f, δ_0, δ_1) for the shear flow problem, *p. 27*

$\mathscr{H}^1$ Space of data for Prandtl at high regularity, *p. 93*

$\mathscr{H}^\sigma$ Space of data for Prandtl at low regularity, *p. 93*

$\mathscr{H}^\sigma_{\alpha,\beta}$ Space of data tuples (5.13) for the vorticity equation (5.12) at regularity σ, *p. 83*

$\mathcal{L}^2_z$ Weighted L^2 space for boundary data with norm (1.21), *p. 20*

Q^1 Solution space $H^{5/3}_x L^2_y \cap L^2_x H^2_y$, *p. 3*

$\mathcal{X}_B$ Hilbert space with norm (1.4) of data triplets (f, δ_0, δ_1) for nonlinear Burgers, *p. 3*

$\mathcal{X}^\sigma$ Banach space of data $(f, \delta_0, \delta_1, \delta_t, \delta_b, v_b)$ with low regularity for the Prandtl problem, defined in (5.8), *p. 80*

$\mathcal{X}^1$ Banach space of data $(f, \delta_0, \delta_1, \delta_t, \delta_b, v_b)$ with higher regularity for the Prandtl problem, defined in (5.9), *p. 80*

$\mathcal{Y}_0$ Notation for $L^2(\Omega)$ during discussions on interpolation, *p. 105*

$\mathcal{Y}_1$ Notation for $H^1_x L^2_y$ with $f_{|\Sigma_0 \cup \Sigma_1} = 0$ during discussions on interpolation, *p. 105*

Z^0 Pagani solution space such that u, $z\partial_x u$ and $\partial_{zz} u$ are L^2, with norm (1.23), *p. 20*

Z^1 Solution space such that $u, \partial_x u \in Z^0$, with norm (1.24), *p. 20*

Z^σ Interpolation space $[Z^0, Z^1]$ for fractional Pagani regularity, *p.* 47

$\mathcal{Z}_B$ Solution space for the Burgers system, *p.* 66

$\mathcal{Z}^1$ Solution space for Prandtl at high regularity, *p.* 93

$\mathcal{Z}^\sigma$ Solution space for Prandtl at low regularity, *p.* 93

Other

γ_b, γ_t Level sets of the unknown solution u of the Prandtl system, *p.* 6

$\overline{\gamma_b}, \overline{\gamma_t}$ Level sets of the function $\mathbb{u}_\mathrm{P}$, *p.* 6

δ_i Boundary data at the inflow boundary Σ_i, *p.* 3

δ_b, δ_t Boundary data for the vorticity on the bottom and top boundaries of Ω_P, *p.* 6

Δ_i Boundary data for $\partial_x u$, given by $\Delta_i = (f + \delta_i'')/z$, see (2.8), *p.* 25

Λ_k Angular profile of the k-th explicit singular solution in the half-plane, *p.* 35

Ξ Shorthand for a data triplet $\Xi = (f, \delta_0, \delta_1)$., *p.* 56

Σ_0 Left inflow boundary $\{x_0\} \times (0, 1)$, see Figure 1.1, *p.* 1

Σ_1 Right inflow boundary $\{x_1\} \times (-1, 0)$, see Figure 1.1, *p.* 1

Σ_i^P Lateral inflow boundaries for the Prandtl system, *p.* 7

$\Upsilon[\delta_i]$ Lateral boundary data for the Burgers system after the change of variables, *p.* 64

$\Upsilon_\mathrm{P}^i[\delta_i]$ Boundary data on Σ_i for the Prandtl system in the new variables, *p.* 79

$\Upsilon_\mathrm{P}^b[\delta_b]$ Boundary data at the bottom for the Prandtl system in the new variables, *p.* 78

$\Upsilon_\mathrm{P}^t[\delta_t]$ Boundary data at the top for the Prandtl system in the new variables, *p.* 78

$\overline{\Phi^j}$ Dual profiles of Lemma 2.7 involved in orthogonality conditions for the shear flow, *p.* 26

χ_i Cut-off function localized near $(x_i, 0)$, *p.* 39

Ω Physical rectangular domain $(x_0, x_1) \times (-1, 1)$, see Figure 1.1, *p.* 1

$\Omega_\pm$ Upper and lower halves of the domain Ω, *p.* 19

Ω_P Physical domain for the resolution of the Prandtl system, *p.* 6

$\overline{f_i}$	Smooth source term associated with the singular solution $\bar{u}^i_{\text{sing}}$, *p.* 40
ℓ^0, ℓ^1	Linear orthogonality conditions of Definition 5.10 for the solvability of Prandtl at high regularity, *p.* 91
ℓ^2	Additional linear form of Definition 5.7 to reconstruct the velocity from the vorticity, *p.* 89
$\overline{\ell^j}$	Linear forms on $\mathcal{H}_K$ giving the orthogonality conditions for the shear flow, *p.* 30
$\bar{M}$	Invertible matrix relating the singular solutions $\bar{u}^i_{\text{sing}}$ with the dual profiles $\overline{\Phi^j}$, *p.* 43
N_B	Nonlinearity associated with the Burgers-type system, *p.* 64
N_P	Nonlinearity associated with the Prandtl system, *p.* 94
r	Radial-like variable given by $r = (z^2 + x^{\frac{2}{3}})^{\frac{1}{2}}$, *p.* 34
r_i	Radial-like variable near $(x_i, 0)$ given by $r_i = (z^2 + \lvert x - x_i\rvert^{\frac{2}{3}})^{\frac{1}{2}}$, *p.* 40
t	Angular-like variable given by $t = zx^{-\frac{1}{3}}$, *p.* 34
t_i	Angular-like variable near $(x_i, 0)$, given by $t_i = (-1)^i z\lvert x - x_i\rvert^{-\frac{1}{3}}$, *p.* 40
$\bar{u}^i_{\text{sing}}$	Reference singular solution localized near $(x_i, 0)$, *p.* 39
$\mathbb{u}_{\mathrm{P}}$	Reference recirculating flow for the Prandtl system, *p.* 6
v_b	Boundary datum for the vertical velocity on the bottom boundary of Ω_P, *p.* 6
v_k	k-th explicit singular solution in the half plane, $v_k = r^{\frac{1}{2}+3k}\Lambda_k(t)$, *p.* 35
$\mathbb{Y}_{\mathrm{P}}$	Inverse function of the reference flow $\mathbb{u}_{\mathrm{P}}$, *p.* 78