Abstract

Fix an arbitrary compact orientable surface with a boundary and consider a uniform bipartite random quadrangulation of this surface with n faces and boundary component lengths of order $\sqrt{n}$ or of lower order. Endow this quadrangulation with the usual graph metric renormalized by $n^{-1/4}$, mark it on each boundary component, and endow it with the counting measure on its vertex set renormalized by n^{-1}, as well as the counting measure on each boundary component renormalized by $n^{-1/2}$. We show that, as n goes to infinity, this random marked measured metric space converges in distribution for the Gromov–Hausdorff–Prokhorov topology, toward a random limiting marked measured metric space called a *Brownian surface*.

This extends known convergence results of uniform random planar quadrangulations with at most one boundary component toward the *Brownian sphere* and toward the *Brownian disk*, by considering the case of quadrangulations on general compact orientable surfaces. Our approach consists in cutting a Brownian surface into elementary pieces that are naturally related to the Brownian sphere and the Brownian disk and their noncompact analogs, the Brownian plane and the Brownian half-plane, and to prove convergence results for these elementary pieces, which are of independent interest.

Keywords: random maps, scaling limits, Brownian sphere, Brownian disk, Brownian surface, Gromov–Hausdorff–Prokhorov topology

Mathematics Subject Classification (2020): 60F17 (primary); 05C10 (secondary)

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