Contents

1 Introduction 1
- 1.1 Context 1
- 1.2 Generalities and terminology on maps 4
- 1.3 The Gromov–Hausdorff–Prokhorov topology 5
- 1.4 The main convergence result 7
- 1.5 Scaling limits of Boltzmann quadrangulations 12
- 1.6 Perspectives 15
- 1.7 Organization of the memoir 16

2 Variants of the Cori–Vauquelin–Schaeffer bijection 17
- 2.1 Basic construction 17
- 2.2 The generalized Chapuy–Marcus–Schaeffer bijection 19
- 2.3 Composite slices 24
- 2.4 Quadrilaterals with geodesic sides 27
- 2.5 Scaling limits of elementary pieces 29

3 Marking and gluing along geodesics 31
- 3.1 Useful facts on the Gromov–Hausdorff–Prokhorov topology and markings 31
- 3.2 Geodesics in metric spaces 34
- 3.3 Gluing along geodesics 36
- 3.4 Proof of Theorem 1.1 41
- 3.5 Gluing quadrangulations from elementary pieces 42
- 3.6 Scaling limit of the collection of elementary pieces 48
- 3.7 Gluing pieces together 51
- 3.8 Topology and Hausdorff dimension 52

4 Convergence of composite slices 55
- 4.1 Metric spaces coded by real functions 55
- 4.2 Random continuum composite slices 61
- 4.3 The Brownian half-plane, and its embedded slices 63
- 4.4 The uniform infinite half-planar quadrangulation 66
- 4.5 Scaling limit of conditioned slices 72

5 Convergence of quadrilaterals with geodesic sides 77
- 5.1 Quadrilaterals coded by two functions 77

5.2 Random continuum quadrilaterals 81
5.3 The Brownian plane, and its embedded quadrilaterals 82
5.4 The uniform infinite planar quadrangulation 87
5.5 Discrete quadrilaterals in the uniform infinite planar quadrangulation . 89
5.6 Scaling limit of conditioned quadrilaterals 94

6 Construction from a continuous unicellular map 99

A Technical lemmas on the Brownian plane 105
A.1 Equivalence of definitions of the Brownian plane 105
A.2 Convergence of the uniform infinite planar quadrangulation to the Brownian plane 106

B Scaling limit of size parameters in labeled maps 111
B.1 Preliminaries 111
B.2 Asymptotics of the scheme 113
B.3 Asymptotics of the size parameters 119
B.4 Boltzmann quadrangulations 121

References 123