References

[1] C. Abraham, Rescaled bipartite planar maps converge to the Brownian map. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016), no. 2, 575–595

[2] R. Abraham, J.-F. Delmas, and P. Hoscheit, A note on the Gromov–Hausdorff–Prokhorov distance between (locally) compact metric measure spaces. *Electron. J. Probab.* **18** (2013), article no. 14

[3] L. Addario-Berry and M. Albenque, The scaling limit of random simple triangulations and random simple quadrangulations. *Ann. Probab.* **45** (2017), no. 5, 2767–2825

[4] L. Addario-Berry and M. Albenque, Convergence of non-bipartite maps via symmetrization of labeled trees. *Ann. H. Lebesgue* **4** (2021), 653–683

[5] L. Addario-Berry and Y. Wen, Joint convergence of random quadrangulations and their cores. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017), no. 4, 1890–1920

[6] M. Albenque and D. Poulalhon, A generic method for bijections between blossoming trees and planar maps. *Electron. J. Combin.* **22** (2015), no. 2, article no. 2.38

[7] D. Aldous, The continuum random tree. I. *Ann. Probab.* **19** (1991), no. 1, 1–28

[8] D. Aldous, The continuum random tree. III. *Ann. Probab.* **21** (1993), no. 1, 248–289

[9] M. Ang, N. Holden, and X. Sun, The SLE loop via conformal welding of quantum disks. *Electron. J. Probab.* **28** (2023), article no. 30

[10] M. Ang, G. Remy, and X. Sun, The moduli of annuli in random conformal geometry. *Ann. Sci. Éc. Norm. Supér. (4)* **58** (2025), no. 4, 1037–1087

[11] O. Angel, B. Kolesnik, and G. Miermont, Stability of geodesics in the Brownian map. *Ann. Probab.* **45** (2017), no. 5, 3451–3479

[12] O. Angel and O. Schramm, Uniform infinite planar triangulations. *Comm. Math. Phys.* **241** (2003), no. 2–3, 191–213

[13] E. Baur, G. Miermont, and G. Ray, Classification of scaling limits of uniform quadrangulations with a boundary. *Ann. Probab.* **47** (2019), no. 6, 3397–3477

[14] E. Baur and L. Richier, Uniform infinite half-planar quadrangulations with skewness. *Electron. J. Probab.* **23** (2018), article no. 54

[15] V. Beffara, C. B. Huynh, and B. Lévêque, Scaling limits for random triangulations on the torus. 2019, arXiv:1905.01873v1

[16] J. Beltran and J.-F. Le Gall, Quadrangulations with no pendant vertices. *Bernoulli* **19** (2013), no. 4, 1150–1175

[17] E. A. Bender and E. R. Canfield, The asymptotic number of rooted maps on a surface. *J. Combin. Theory Ser. A* **43** (1986), no. 2, 244–257

[18] J. Bertoin, N. Curien, and I. Kortchemski, Random planar maps and growth-fragmentations. *Ann. Probab.* **46** (2018), no. 1, 207–260

[19] J. Bettinelli, Scaling limits for random quadrangulations of positive genus. *Electron. J. Probab.* **15** (2010), 1594–1644

[20] J. Bettinelli, The topology of scaling limits of positive genus random quadrangulations. *Ann. Probab.* **40** (2012), no. 5, 1897–1944

[21] J. Bettinelli, Scaling limit of random planar quadrangulations with a boundary. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015), no. 2, 432–477

[22] J. Bettinelli, Geodesics in Brownian surfaces (Brownian maps). *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016), no. 2, 612–646

[23] J. Bettinelli, A bijection for nonorientable general maps. *Ann. Inst. Henri Poincaré D* **9** (2022), no. 4, 733–791

[24] J. Bettinelli, E. Jacob, and G. Miermont, The scaling limit of uniform random plane maps, *via* the Ambjørn–Budd bijection. *Electron. J. Probab.* **19** (2014), article no. 74

[25] J. Bettinelli and G. Miermont, Compact Brownian surfaces I: Brownian disks. *Probab. Theory Related Fields* **167** (2017), no. 3–4, 555–614

[26] N. H. Bingham, C. M. Goldie, and J. L. Teugels, *Regular variation*. Encyclopedia Math. Appl. 27, Cambridge University Press, Cambridge, 1989

[27] J. Bouttier, Planar maps and random partitions. 2019, Habilitation thesis, Université Paris-Sud, https://tel.archives-ouvertes.fr/tel-02417269, visited on 10 March 2026

[28] J. Bouttier, P. Di Francesco, and E. Guitter, Planar maps as labeled mobiles. *Electron. J. Combin.* **11** (2004), no. 1, article no. 69

[29] J. Bouttier and E. Guitter, Distance statistics in quadrangulations with a boundary, or with a self-avoiding loop. *J. Phys. A* **42** (2009), no. 46, article no. 465208

[30] J. Bouttier and E. Guitter, Planar maps and continued fractions. *Comm. Math. Phys.* **309** (2012), no. 3, 623–662

[31] M. R. Bridson and A. Haefliger, *Metric spaces of non-positive curvature*. Grundlehren Math. Wiss. 319, Springer, Berlin, 1999

[32] D. Burago, Y. Burago, and S. Ivanov, *A course in metric geometry*. Grad. Stud. Math. 33, American Mathematical Society, Providence, RI, 2001

[33] A. Caraceni and N. Curien, Geometry of the uniform infinite half-planar quadrangulation. *Random Structures Algorithms* **52** (2018), no. 3, 454–494

[34] G. Chapuy and M. Dołęga, A bijection for rooted maps on general surfaces. *J. Combin. Theory Ser. A* **145** (2017), 252–307

[35] G. Chapuy, M. Marcus, and G. Schaeffer, A bijection for rooted maps on orientable surfaces. *SIAM J. Discrete Math.* **23** (2009), no. 3, 1587–1611

[36] P. Chassaing and B. Durhuus, Local limit of labeled trees and expected volume growth in a random quadrangulation. *Ann. Probab.* **34** (2006), no. 3, 879–917

[37] P. Chassaing and G. Schaeffer, Random planar lattices and integrated superBrownian excursion. *Probab. Theory Related Fields* **128** (2004), no. 2, 161–212

[38] R. Cori and B. Vauquelin, Planar maps are well labeled trees. *Canad. J. Math.* **33** (1981), no. 5, 1023–1042

[39] N. Curien, *Peeling random planar maps. École d'Été de Probabilités de Saint-Flour XLIX – 2019*. Lecture Notes in Math. 2335, Springer, Cham, 2023

[40] N. Curien and J.-F. Le Gall, The Brownian plane. *J. Theoret. Probab.* **27** (2014), no. 4, 1249–1291

[41] N. Curien and J.-F. Le Gall, First-passage percolation and local modifications of distances in random triangulations. *Ann. Sci. Éc. Norm. Supér. (4)* **52** (2019), no. 3, 631–701

[42] N. Curien, L. Ménard, and G. Miermont, A view from infinity of the uniform infinite planar quadrangulation. *ALEA Lat. Am. J. Probab. Math. Stat.* **10** (2013), no. 1, 45–88

[43] N. Curien and G. Miermont, Uniform infinite planar quadrangulations with a boundary. *Random Structures Algorithms* **47** (2015), no. 1, 30–58

[44] F. David, Planar diagrams, two-dimensional lattice gravity and surface models. *Nuclear Phys. B* **257** (1985), no. 1, 45–58

[45] F. David, A. Kupiainen, R. Rhodes, and V. Vargas, Liouville quantum gravity on the Riemann sphere. *Comm. Math. Phys.* **342** (2016), no. 3, 869–907

[46] J. Ding, J. Dubédat, A. Dunlap, and H. Falconet, Tightness of Liouville first passage percolation for $\gamma \in (0, 2)$. *Publ. Math. Inst. Hautes Études Sci.* **132** (2020), 353–403

[47] J. Ding, J. Dubédat, and E. Gwynne, Introduction to the Liouville quantum gravity metric. In *ICM—International Congress of Mathematicians. Vol. 6. Sections 12–14*, pp. 4212–4244, European Mathematical Society, Berlin, 2023

[48] B. Eynard, *Counting surfaces. CRM Aisenstadt Chair lectures*. Prog. Math. Phys. 70, Birkhäuser, Cham, 2016

[49] C. Guillarmou, A. Kupiainen, R. Rhodes, and V. Vargas, Segal's axioms and bootstrap for Liouville theory. 2025, arXiv:2112.14859v2

[50] C. Guillarmou, R. Rhodes, and V. Vargas, Polyakov's formulation of $2d$ bosonic string theory. *Publ. Math. Inst. Hautes Études Sci.* **130** (2019), 111–185

[51] E. Gwynne, N. Holden, and X. Sun, Mating of trees for random planar maps and Liouville quantum gravity: a survey. In *Topics in statistical mechanics*, pp. 41–120, Panor. Synthèses 59, Société Mathématique de France, Paris, 2023

[52] E. Gwynne and J. Miller, Scaling limit of the uniform infinite half-plane quadrangulation in the Gromov–Hausdorff–Prokhorov-uniform topology. *Electron. J. Probab.* **22** (2017), article no. 84

[53] E. Gwynne and J. Miller, Metric gluing of Brownian and $\sqrt{8/3}$-Liouville quantum gravity surfaces. *Ann. Probab.* **47** (2019), no. 4, 2303–2358

[54] E. Gwynne and J. Miller, Convergence of the self-avoiding walk on random quadrangulations to $\mathrm{SLE}_{8/3}$ on $\sqrt{8/3}$-Liouville quantum gravity. *Ann. Sci. Éc. Norm. Supér. (4)* **54** (2021), no. 2, 305–405

[55] E. Gwynne and J. Miller, Existence and uniqueness of the Liouville quantum gravity metric for $\gamma \in (0, 2)$. *Invent. Math.* **223** (2021), no. 1, 213–333

[56] E. Gwynne, J. Miller, and S. Sheffield, The Tutte embedding of the Poisson–Voronoi tessellation of the Brownian disk converges to $\sqrt{8/3}$-Liouville quantum gravity. *Comm. Math. Phys.* **374** (2020), no. 2, 735–784

[57] E. Gwynne, J. Miller, and S. Sheffield, An invariance principle for ergodic scale-free random environments. *Acta Math.* **228** (2022), no. 2, 303–384

[58] N. Holden and X. Sun, Convergence of uniform triangulations under the Cardy embedding. *Acta Math.* **230** (2023), no. 1, 93–203

[59] V. G. Knizhnik, A. M. Polyakov, and A. B. Zamolodchikov, Fractal structure of 2D-quantum gravity. *Modern Phys. Lett. A* **3** (1988), no. 8, 819–826

[60] M. Krikun, Local structure of random quadrangulations. 2006, arXiv:math/0512304v2

[61] A. Kupiainen, R. Rhodes, and V. Vargas, Integrability of Liouville theory: proof of the DOZZ formula. *Ann. of Math. (2)* **191** (2020), no. 1, 81–166

[62] S. K. Lando and A. K. Zvonkin, *Graphs on surfaces and their applications (with an appendix by Don B. Zagier)*. Encyclopaedia Math. Sci. 141, Springer, Berlin, 2004

[63] J.-F. Le Gall, *Spatial branching processes, random snakes and partial differential equations*. Lect. Math. ETH Zürich, Birkhäuser, Basel, 1999

[64] J.-F. Le Gall, A conditional limit theorem for tree-indexed random walk. *Stochastic Process. Appl.* **116** (2006), no. 4, 539–567

[65] J.-F. Le Gall, The topological structure of scaling limits of large planar maps. *Invent. Math.* **169** (2007), no. 3, 621–670

[66] J.-F. Le Gall, Geodesics in large planar maps and in the Brownian map. *Acta Math.* **205** (2010), no. 2, 287–360

[67] J.-F. Le Gall, Uniqueness and universality of the Brownian map. *Ann. Probab.* **41** (2013), no. 4, 2880–2960

[68] J.-F. Le Gall, Brownian disks and the Brownian snake. *Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019), no. 1, 237–313

[69] J.-F. Le Gall, Brownian geometry. *Jpn. J. Math.* **14** (2019), no. 2, 135–174

[70] J.-F. Le Gall, The Brownian disk viewed from a boundary point. *Ann. Inst. Henri Poincaré Probab. Stat.* **58** (2022), no. 2, 1091–1119

[71] J.-F. Le Gall, Geodesic stars in random geometry. *Ann. Probab.* **50** (2022), no. 3, 1013–1058

[72] J.-F. Le Gall and G. Miermont, Scaling limits of random planar maps with large faces. *Ann. Probab.* **39** (2011), no. 1, 1–69

[73] J.-F. Le Gall and F. Paulin, Scaling limits of bipartite planar maps are homeomorphic to the 2-sphere. *Geom. Funct. Anal.* **18** (2008), no. 3, 893–918

[74] J.-F. Le Gall and A. Riera, Growth-fragmentation processes in Brownian motion indexed by the Brownian tree. *Ann. Probab.* **48** (2020), no. 4, 1742–1784

[75] J.-F. Le Gall and A. Riera, Spine representations for non-compact models of random geometry. *Probab. Theory Related Fields* **181** (2021), no. 1–3, 571–645

[76] J.-F. Marckert and A. Mokkadem, The depth first processes of Galton–Watson trees converge to the same Brownian excursion. *Ann. Probab.* **31** (2003), no. 3, 1655–1678

[77] C. Marzouk, On scaling limits of random trees and maps with a prescribed degree sequence. *Ann. H. Lebesgue* **5** (2022), 317–386

[78] G. Miermont, On the sphericity of scaling limits of random planar quadrangulations. *Electron. Commun. Probab.* **13** (2008), 248–257

[79] G. Miermont, Tessellations of random maps of arbitrary genus. *Ann. Sci. Éc. Norm. Supér. (4)* **42** (2009), no. 5, 725–781

[80] G. Miermont, The Brownian map is the scaling limit of uniform random plane quadrangulations. *Acta Math.* **210** (2013), no. 2, 319–401

[81] J. Miller and W. Qian, Geodesics in the Brownian map: Strong confluence and geometric structure. *Mem. Amer. Math. Soc.* **315** (2025), no. 1602, vi+117 pp.

[82] J. Miller and S. Sheffield, Liouville quantum gravity and the Brownian map I: the QLE(8/3, 0) metric. *Invent. Math.* **219** (2020), no. 1, 75–152

[83] J. Miller and S. Sheffield, An axiomatic characterization of the Brownian map. *J. Éc. Polytech. Math.* **8** (2021), 609–731

[84] J. Miller and S. Sheffield, Liouville quantum gravity and the Brownian map II: Geodesics and continuity of the embedding. *Ann. Probab.* **49** (2021), no. 6, 2732–2829

[85] J. Miller and S. Sheffield, Liouville quantum gravity and the Brownian map III: the conformal structure is determined. *Probab. Theory Related Fields* **179** (2021), no. 3–4, 1183–1211

[86] V. V. Petrov, *Sums of independent random variables*. Ergeb. Math. Grenzgeb. 82, Springer, New York, 1975

[87] J. W. Pitman, One-dimensional Brownian motion and the three-dimensional Bessel process. *Adv. in Appl. Probab.* **7** (1975), no. 3, 511–526

[88] A. M. Polyakov, Quantum geometry of bosonic strings. *Phys. Lett. B* **103** (1981), no. 3, 207–210

[89] L. B. Richmond and N. C. Wormald, Almost all maps are asymmetric. *J. Combin. Theory Ser. B* **63** (1995), no. 1, 1–7

[90] G. Schaeffer, *Conjugaison d'arbres et cartes combinatoires aléatoires*. Ph.D. thesis, 1998, Université Bordeaux

[91] S. Sheffield, What is a random surface? In *ICM—International Congress of Mathematicians. Vol. 2. Plenary lectures*, pp. 1202–1258, European Mathematical Society, Berlin, 2023

[92] D. W. Stroock, *Probability theory. An analytic view*. 2nd edn., Cambridge University Press, Cambridge, 2011

[93] C. Villani, *Optimal transport: Old and new*. Grundlehren Math. Wiss. 338, Springer, Berlin, 2009

[94] B. Wu, Conformal bootstrap on the annulus in Liouville CFT. 2024, arXiv:2203.11830v5