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Károly J. Böröczky

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**Isoperimetric Inequalities,  
Brunn–Minkowski Theory  
and Minkowski-Type  
Monge–Ampère Equations  
on the Sphere**



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