

Abstract

This memoir introduces a new algebraic notion: that of a *triangulated persistence category* (TPC), which refines the notion of a triangulated category in the same sense that a persistence module refines that of a vector space. The spaces of morphisms of such a TPC are persistence modules, and the category is endowed with a class of weighted distinguished triangles. Under favourable conditions, we show that the derived Fukaya category admits a TPC refinement, and we apply this to deduce a global rigidity result for spaces of compact, exact Lagrangians in certain Liouville manifolds: we construct a metric on this space with intrinsic symplectic properties.

Keywords. triangulated category, persistence module, symplectic manifold, Lagrangian submanifold, Floer homology, Fukaya category

Mathematics Subject Classification (2020): 53D12 (primary); 53D37, 55N31 (secondary)

Acknowledgments. The third author is grateful to Mike Usher for useful discussions. We thank Leonid Polterovich for mentioning to us the work of Fan–Filip [30]. We thank the referee of the first version of this memoir for pointing out the relation between our algebraic construction and Verdier localization. We are also grateful to a second referee for a very careful and critical reading of a later version, for pointing out several inaccuracies, and for making very useful suggestions that helped to improve the exposition. We thank a third referee for pointing out the paper [51] (see also Remark 2.20).

Funding. The second author was supported by an individual NSERC Discovery grant. Part of this work was completed while the third author held a CRM–ISM Postdoctoral Research Fellowship at the *Centre de recherches mathématiques* in Montréal. He thanks the institute for its warm hospitality. The third author is partially supported by the National Key R&D Program of China (No. 2023YFA1010500), the NSFC (Nos. 12301081 and 12361141812), and the USTC Research Funds of the Double First-Class Initiative.