Introduction

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Background and scope

Complex systems formed by many interacting agents are ubiquitous in the natural and social sciences. They account for phenomena as diverse as the motion of gas atoms, the emergence of collective behaviours in biological populations, the organisation of socio-economic systems, just to name a few. A fundamental paradigm in the study of these systems is the transition from microscopic to macroscopic scales: instead of following individual agents, one models the population density, which in the many-particle limit is commonly described by mean-field or kinetic equations and, at larger scales, by hydrodynamic models. Originating in statistical physics, these ideas now underpin models across disciplines collective motion and swarming, chemotaxis and population dynamics, traffic and pedestrian flow, opinion dynamics, economics and finance, and epidemiology.

There are several challenges involved in modelling complex systems. On the theoretical side, one must rigorously validate the limit processes that connect microscopic, mesoscopic, and macroscopic scales. On the computational side, efficient methods must be devised for simulating extremely large systems, often through suitable approximations, coupling different scales. On the applied side, a crucial goal is the design of strategies for influencing or controlling collective dynamics in order to achieve desired outcomes. These questions drive modern mathematical research at the interface of modelling, analysis, probability, computation, and control.

Central among these directions are the derivation of mean-field limits and Boltzmann-type equations, which can provide rigorous formalisms to justify the passage from microscopic interacting particle systems to kinetic models. On larger space-time scales, the resulting mesoscopic equations further give rise, upon equilibrium closure, to fluid-like or density-based PDEs that describe emergent macroscopic flows. In such

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2020 Mathematics Subject Classification: 82C22 (primary); 35A35, 35L65, 49J20, 65Mxx, 93Cxx, 35Q20, 35B40 (secondary).
Keywords: multi-agent systems; kinetic equations; hydrodynamic limits; controllability; nonlocal interactions; uncertainty quantification; optimisation; machine learning.

a hierarchical derivation, the role of *non-local interactions* has become increasingly prominent, as more realistic models of collective behaviour rely on communication mechanisms that are not purely metric, but instead depend on ranks, networks, or anisotropic perception. This also calls for a careful study of suitable classes of interaction kernels: such classes must be broad enough to capture realistic behaviours, and yet contain suitable functions so that mean-field and hydrodynamic limits can be established rigorously.

Beyond analytic advances, these models now drive a wide range of applications. In public health they underpin epidemic forecasting and intervention design; in the life sciences they capture collective cell migration, chemotaxis, and population dispersal; in infrastructure and mobility they describe traffic and pedestrian dynamics; in socio-economic contexts they model opinion formation, polarisation, and radicalisation. They also guide the coordination of robotic swarms and autonomous systems, where micro-to-macro reasoning supports scalable strategies under safety and resource constraints. Across these domains, the multiscale viewpoint links measurable, agent-level mechanisms to macroscopic behaviour.

Strongly related to applications, research lines where the design of control mechanisms and optimality principles that rule the agents' dynamics are investigated have become of paramount importance. For instance, how external interventions, localised inputs, or structured feedback can steer the global evolution of large systems, as well as how equilibria emerge from collective optimisation of individuals' decisions. These perspectives connect, on one side, with *mean-field control*, where the focus is on the optimal centralized control of population distributions. On the other side, they connect with *mean-field games*, which are closely related yet game-theoretic in nature. Mean-field games arise as the mean-field limit of many agents, each solving their own payoff maximisation problem and interacting through the population state, leading to mean-field Nash equilibria.

Another active direction concerns the numerical approximation of these problems across scales. *Asymptotic-preserving* and *structure-preserving* schemes are particularly important, since they can handle disparate regimes while remaining consistent with the limiting dynamics and maintaining the fundamental invariants (e.g., mass, positivity, momentum, energy/entropy) that must be preserved across scales. Furthermore, coping with noisy and partial observations together with large, high-dimensional parameter spaces demand probabilistic modelling and robust, scalable algorithms. In this context for multiscale problems, uncertainty quantification (UQ) focusses on *scale-bridging* propagation of uncertainty, data-driven calibration and identifiability (forward/inverse UQ), and sensitivity analysis to rank high-dimensional parameters. In practice, multi-fidelity and multi-level Monte Carlo methods, combined with surrogate or reduced-order models, yield tractable estimators and enable risk-aware, robust control under noise and model misspecification.

Furthermore, based on these research directions, new bridges are now being built between *multi-agent modelling* and *machine learning* or *artificial intelligence*. Interacting particle systems inspire algorithms for optimisation, sampling, and data analysis, while AI tools provide powerful means for identification, calibration, and control of large-scale models from data. This cross-fertilisation is rapidly expanding the scope of the field, with applications ranging from reinforcement learning methods informed by collective dynamics to data-driven discovery of governing equations.

Topics covered in the volume

The chapters in this volume reflect many of the themes outlined above, grouped around derivation of micro-to-macro models; computational methods and applications.

The volume starts with results related to the *derivation and analysis across scales*, the work includes rigorous passage from particle models to kinetic, and further to macroscopic, PDEs. The topics covered include synchronisation phenomena in oscillator populations, mean-field convergence in swarming models, propagation of chaos in flocking systems with topological interactions, the study of dissipative traffic flow models and their connection with alignment dynamics, and the controllability of particles in viscous fluids bridging microscopic hydrodynamics with kinetic and macroscopic descriptions.

The *algorithmic and computational dimension* of multi-agent systems is represented too. The volume includes work on uncertainty quantification for kinetic traffic systems through multi-fidelity and multi-level Monte Carlo approaches, unifying frameworks for meta-heuristic optimisation inspired by interacting particle systems, and novel consensus-based optimisation schemes enriched with genetic dynamics to efficiently handle multi-modal problems.

Finally, applications to *biology and society* in multi-agent systems constitute a further central focus. The chapters on this theme explore the emergence of patterns in chemotaxis models with nonlinear diffusion, particle methods for congested dynamics governed by nonlocal balance laws, epidemic models and models of social phenomena such as opinion formation or radicalisation, blending mechanistic modelling with data-driven perspectives.

Altogether, the volume demonstrates the richness of multi-agent systems across scales and domains: it encompasses rigorous theoretical analysis, advanced numerical methods and provides applications of immediate relevance to various fields of research.