
PRESS

EMS Series of Lectures in Mathematics

Edited by Ari Laptev (Imperial College, London, UK)

The *EMS Series of Lectures in Mathematics* is a book series aimed at students, professional mathematicians and scientists. It publishes polished notes arising from seminars or lecture series in all fields of pure and applied mathematics, including the reissue of classic texts of continuing interest. The individual volumes are intended to give a rapid and accessible introduction into their particular subject, guiding the audience to topics of current research and the more advanced and specialized literature.

Previously published in this series:

K. Wehrheim, *Uhlenbeck Compactness*
T. Ekedahl, *One Semester of Elliptic Curves*
S. V. Matveev, *Lectures on Algebraic Topology*
J. C. Várilly, *An Introduction to Noncommutative Geometry*
R. Müller, *Differential Harnack Inequalities and the Ricci Flow*
E. del Barrio, P. Deheuvels, S. van de Geer, *Lectures on Empirical Processes*
I. A. Taimanov, *Lectures on Differential Geometry*
M. J. Mohlenkamp, M. C. Pereyra, *Wavelets, Their Friends, and What They Can Do for You*
S. E. Payne, J. A. Thas, *Finite Generalized Quadrangles*
H. Holden, K. H. Karlsen, K.-A. Lie, N. H. Risebro, *Splitting Methods for Partial Differential Equations with Rough Solutions*
K. Harada, *"Moonshine" of Finite Groups*
Y. A. Neretin, *Lectures on Gaussian Integral Operators and Classical Groups*
D. Calaque, C. A. Rossi, *Lectures on Duflo Isomorphisms in Lie Algebra and Complex Geometry*
C. Carmeli, L. Caston, R. Fioresi, *Mathematical Foundations of Supersymmetry*
H. Triebel, *Faber Systems and Their Use in Sampling, Discrepancy, Numerical Integration*
K. Thas, *A Course on Elation Quadrangles*
M. Khalkhali, *Basic Noncommutative Geometry*
B. Grébert, T. Kappeler, *The Defocusing NLS Equation and Its Normal Form*
A. Sergeev, *Lectures on Universal Teichmüller Space*
M. Aschenbrenner, S. Friedl, H. Wilton, *3-Manifold Groups*
H. Triebel, *Tempered Homogeneous Function Spaces*
K. Bringmann, Y. Bugeaud, T. Hilberdink, J. Sander, *Four Faces of Number Theory*
A. Cavicchioli, F. Hegenbarth, D. Repovš, *Higher-Dimensional Generalized Manifolds: Surgery and Constructions*
D. Barilari, U. Boscain, M. Sigalotti (Eds.), *Geometry, Analysis and Dynamics on sub-Riemannian Manifolds, Volume I and II*
F. Dal'Bo, F. Ledrappier, A. Wilkinson (Eds.), *Dynamics Done with Your Bare Hands*
H. Triebel, *PDE Models for Chemotaxis and Hydrodynamics in Supercritical Function Spaces*
F. Michel, C. Weber, *Higher-Dimensional Knots According to Michel Kervaire*
R. Kessar, G. Malle, D. Testerman (Eds.), *Local Representation Theory and Simple Groups*
H. Triebel, *Function Spaces with Dominating Mixed Smoothness*
C. Ciliberto, *Classification of Complex Algebraic Surfaces*
F. Hiai, *Lectures on Selected Topics in von Neumann Algebras*
J. C. García-Ardila, F. Marcellán, M. E. Marriaga, *Orthogonal Polynomials and Linear Functionals*
N. Berglund, *An Introduction to Singular Stochastic PDEs*
R. Speicher, *Lectures on Random Matrices*
U. Zannier, *Lecture Notes on Diophantine Analysis* (2nd ed.)

Applications of Noncommutative Geometry to Gauge Theories, Field Theories, and Quantum Space-Time

Edited by
Ali H. Chamseddine
Alain Connes
Masoud Khalkhali
Joseph Kouneiher

EMS PRESS

Editors

Ali H. Chamseddine
Department of Physics
American University of Beirut
P.O. Box 11-0236
1107 2020 Beirut, Lebanon
Email: chams@aub.edu.lb

Masoud Khalkhali
Department of Mathematics
University of Western Ontario
Middlesex College
London ON N6A 5B7, Canada
Email: masoud@uwo.ca

Alain Connes
Collège de France
3 Rue d'Ulm
75005 Paris, France
Email: alain@connes.org

Joseph Kouneiher
Sciences and Technologies Department / INSPE
Université Côte d'Azur
102 Parvis Gillet, Avenue Philippe Seguin
83300 Draguignan, France
Email: joseph.kouneiher@univ-cotedazur.fr

Mathematics Subject Classification 2020: 58B34 (primary); 14A22, 16S38, 46L87, 81R60, 81T13, 81T75, 83C65 (secondary).

Keywords: noncommutative geometry, spectral geometry, spectral triples, space-time, gauge theories, quantum field theory, standard model, spectral action principle, Dirac operators, quantum gravity

ISBN 978-3-98547-108-9, eISBN 978-3-98547-608-4, DOI 10.4171/ELM/37

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at http://dnb.dnb.de.

Published by EMS Press, an imprint of the

European Mathematical Society – EMS – Publishing House GmbH
Institut für Mathematik
Technische Universität Berlin
Straße des 17. Juni 136
10623 Berlin, Germany

https://ems.press

© 2026 EMS Press

Typeset by AfricanType, Cairo, Egypt
Printed in Germany
♾ Printed on acid free paper