Contents

Preface v

Contents vii

1 Introduction 1
by Alain Connes, Joseph Kouneiher, Ali H. Chamseddine, and Masoud Khalkhali

General perspective 1
Zeta spectral triples 1
Hearing the shape of the universe 3
Bootstrapping noncommutative geometry with Dirac ensembles . 6
Symmetric spaces, non-formal star products and Drinfel'd twists . 8
Noncommutative geometry of gravity, strings and fields 9

2 The spectral approach as an organizing principle of the laws of nature .. 13
by Joseph Kouneiher

1 Spectrality and physical law 13
2 Geometry from spectral data 15
3 Cohomology as the semantics of spectra 19
4 Gauge fields and dynamics as spectral fluctuations 21
5 Spectral truncation, information, and renormalization 24
6 Perspectives: Spectral universality and quantum space-time 35
References 37

3 Zeta spectral triples 39
by Alain Connes, Caterina Consani, and Henri Moscovici

1 Introduction 39
2 Preliminaries 41
3 The Weil quadratic form QW 43
4 The matrix of QW_λ in the basis V_n 50
5 The infrared spectral triples 55
6 Numerical results 66
7 Outlook 68
8 The missing steps 74
References 75

4 Hearing the shape of the universe: A personal journey in noncommutative geometry 77
by Ali H. Chamseddine
1 Introduction 77
2 Early attempts 79
3 The spectral action principle 87
4 Scale invariance and the dilaton in the spectral action 94
5 Including the right-handed neutrinos 98
6 Fermionic Hilbert space, chirality, and gauge representations 104
7 Phenomenology and predictions 106
8 Why the Standard Model? 108
9 Quantum gravity boundary terms from NCG 110
10 NCG as framework for unification: Tensorial notation 113
11 Resilience of the spectral SM 120
12 Parity violation in the spectral action 124
13 The uncanny precision of the spectral action 125
14 Away with order one condition 128
15 Emergence of the Pati–Salam model 130
16 Grand unification in the Pati–Salam model 136
17 Quanta of geometry: Towards quantum gravity 138
18 Entropy and the spectral action 145
19 Spectral action in matrix form 149
20 Conclusion 152
References 155

5 Bootstrapping noncommutative geometry with Dirac ensembles 159
by Masoud Khalkhali and Nathan Pagliaroli
1 Introduction 159
2 Origins of the bootstrap program 161
3 Dirac ensembles 163
4 Bootstrapping Dirac ensembles via positivity 172
5 Bootstrapping multimatrix quantum mechanics 184
6 A tale of two cities 186
7 Bootstrap methods in spectral geometry 202
8 Future directions 205
References 208

6 Symmetric spaces, non-formal star products and Drinfel'd twists 213
by Pierre Bieliavsky
1 Introduction 213
2 Symplectic symmetric spaces 214
3 The retract method in formal deformation quantization 231
4 Admissible phases, Weinstein's and Ševera's areas 247
5 Oscillatory integrals on exponential Lie groups 269
6 Left-invariant non-formal star-products and Drinfel'd–Iwasawa twists 279
7 Quantization of the hyperbolic plane and Drinfel'd twists 290
A André and Julianne Unterberger's symbolic calculus 311
References 312

7 Noncommutative geometry of gravity, strings and fields: A panoramic overview 315
by Richard J. Szabo
1 Introduction 315
2 Spacetime quantisation 318
3 Mathematical basis of quantum geometry 323
4 Quantum theory on quantised spacetime 330
5 Noncommutative geometry in string theory 336
6 Deformation quantisation 348
7 Noncommutative renormalisation 356
8 Yang–Mills matrix models and fuzzy field theories 366
References 377

List of contributors 389