Abstract

In this memoir, we develop a theory of bordered HF^- using the link surgery formula of Manolescu and Ozsváth. We interpret their link surgery complexes as type-D modules over an associative algebra $\mathcal{K}$, which we introduce. We prove a connected sum formula, which we interpret as an A_∞-tensor product over our algebra $\mathcal{K}$. Topologically, this connected sum formula may be viewed as a formula for gluing along torus boundary components.

We discuss several important examples. As a first example, if K_1 and K_2 are knots in S^3, and Y is obtained by gluing the complements of K_1 and K_2 together using an orientation reversing diffeomorphism of their boundaries, then our theory may be used to compute $\mathrm{CF}^-(Y)$ from $\mathrm{CFK}^\infty(K_1)$ and $\mathrm{CFK}^\infty(K_2)$. By computing the type-D modules for rationally framed solid tori, our theory gives a version of the link surgery formula for rationally framed links. As a final example, we use our theory to derive the Heegaard Floer homology of all 3-manifolds which bound the plumbing of a tree of disk bundles over 2-spheres.

Keywords: Heegaard Floer homology, Dehn surgery, link surgery, link Floer homology, link surgery complex, bordered Heegaard Floer homology

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