

## References

- [1] O. Beznosova and A. Reznikov, [Sharp estimates involving  \$A\_\infty\$  and  \$L \log L\$  constants, and their applications to PDE](#) (in Russian). *Algebra i Analiz* **26** (2014), no. 1, 40–67 [English translation: \*St. Petersburg Math. J.\* \*\*26\*\* \(2015\), no. 1, 27–47](#)
- [2] B. Bojarski, C. Sbordone, and I. Wik, [The Muckenhoupt class  \$A\_1\(\mathbf{R}\)\$](#) . *Studia Math.* **101** (1992), no. 2, 155–163
- [3] A. Burchard, G. Dafni, and R. Gibara, [Mean oscillation bounds on rearrangements](#). *Trans. Amer. Math. Soc.* **375** (2022), no. 6, 4429–4444
- [4] D. L. Burkholder, [Boundary value problems and sharp inequalities for martingale transforms](#). *Ann. Probab.* **12** (1984), no. 3, 647–702
- [5] L. Caffarelli, L. Nirenberg, and J. Spruck, [The Dirichlet problem for the degenerate Monge–Ampère equation](#). *Rev. Mat. Iberoam.* **2** (1986), no. 1-2, 19–27
- [6] M. Cwikel, Y. Sagher, and P. Shvartsman, [A new look at the John–Nirenberg and John–Strömberg theorems for BMO](#). *J. Funct. Anal.* **263** (2012), no. 1, 129–166
- [7] M. Dindoš and T. Wall, [The sharp  \$A\_p\$  constant for weights in a reverse–Hölder class](#). *Rev. Mat. Iberoam.* **25** (2009), no. 2, 559–594
- [8] E. Dobronravov, [A sharp symmetric integral form of the John–Nirenberg inequality](#). *Proc. Amer. Math. Soc.* **152** (2024), no. 5, 2087–2101
- [9] M. Ghomi, [Boundary torsion and convex caps of locally convex surfaces](#). *J. Differential Geom.* **105** (2017), no. 3, 427–487
- [10] B. Guan, [The Dirichlet problem for Monge–Ampère equations in non-convex domains and spacelike hypersurfaces of constant Gauss curvature](#). *Trans. Amer. Math. Soc.* **350** (1998), no. 12, 4955–4971
- [11] H. Hedenmalm, D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, [Sharpening Hölder’s inequality](#). *J. Funct. Anal.* **275** (2018), no. 5, 1280–1319
- [12] S. V. Hruščev, [A description of weights satisfying the  \$A\_\infty\$  condition of Muckenhoupt](#). *Proc. Amer. Math. Soc.* **90** (1984), no. 2, 253–257
- [13] P. Ivanisvili, [Inequality for Burkholder’s martingale transform](#). *Anal. PDE* **8** (2015), no. 4, 765–806
- [14] P. Ivanishvili, N. N. Osipov, D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, [On Bellman function for extremal problems in BMO](#). *C. R. Math. Acad. Sci. Paris* **350** (2012), no. 11-12, 561–564
- [15] P. Ivanisvili, N. N. Osipov, D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, [Sharp estimates of integral functionals on classes of functions with small mean oscillation](#). *C. R. Math. Acad. Sci. Paris* **353** (2015), no. 12, 1081–1085
- [16] P. Ivanisvili, N. N. Osipov, D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, [Bellman function for extremal problems in BMO](#). *Trans. Amer. Math. Soc.* **368** (2016), no. 5, 3415–3468

- [17] P. Ivanisvili, D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, [Bellman function for extremal problems in BMO II: Evolution](#). *Mem. Amer. Math. Soc.* **255** (2018), no. 1220
- [18] Ł. Kamiński and A. Osękowski, [Sharp Lorentz-norm estimates for BMO martingales](#). *Statist. Probab. Lett.* **173** (2021), article no. 109068
- [19] S. Karlin and W. J. Studden, *Tchebycheff systems: With applications in analysis and statistics*. Pure Appl. Math. XV, Interscience Publishers John Wiley & Sons, New York-London-Sydney, 1966
- [20] J. H. B. Kemperman, [The general moment problem, a geometric approach](#). *Ann. Math. Statist.* **39** (1968), 93–122
- [21] I. Klemes, *I. Idempotent multipliers of  $H^1$  on the circle. II. A mean oscillation inequality for rearrangements*. Ph.D. thesis, California Institute of Technology, 1985, <https://thesis.caltech.edu/11349/> visited on July 1, 2026
- [22] I. Klemes, [A mean oscillation inequality](#). *Proc. Amer. Math. Soc.* **93** (1985), no. 3, 497–500
- [23] A. A. Korenovskii, [On the connection between mean oscillation and exact integrability classes of functions](#) (in Russian). *Mat. Sb.* **181** (1990), no. 12, 1721–1727. English translation: *Math. USSR-Sb.* **71** (1992), no. 2, 561–567
- [24] A. A. Korenovskii, [The exact continuation of a reverse Hölder inequality and Muckenhoupt's conditions](#) (in Russian). *Mat. Zametki* **52** (1992), no. 6, 32–44. English translation: *Math. Notes* **52** (1992), no. 6, 1192–1201
- [25] M. G. Kreĭn and A. A. Nudel'man, *The Markov moment problem and extremal problems*. Transl. Math. Monogr. 50, American Mathematical Society, Providence, RI, 1977
- [26] N. V. Krylov, [Smoothness of the value function for a controlled diffusion process in a domain](#). *Izv. Akad. Nauk SSSR Ser. Mat.* **53** (1989), no. 1, 66–96 (in Russian). English translation: *Math. USSR-Izv.* **34** (1990), no. 1, 65–95
- [27] A. K. Lerner, [The John–Nirenberg inequality with sharp constants](#). *C. R. Math. Acad. Sci. Paris* **351** (2013), no. 11–12, 463–466
- [28] F. Nazarov, S. Treil, and A. Volberg, [Bellman function in stochastic control and harmonic analysis](#). In *Systems, approximation, singular integral operators, and related topics (Bordeaux, 2000)*, pp. 393–423, Oper. Theory Adv. Appl. 129, Birkhäuser, Basel, 2001
- [29] F. L. Nazarov and S. R. Treil', [The hunt for a Bellman function: applications to estimates for singular integral operators and to other classical problems of harmonic analysis](#) (in Russian). *Algebra i Analiz* **8** (1996), no. 5, 32–162. English translation: *St. Petersburg Math. J.* **8** (1997), no. 5, 721–824
- [30] A. Osękowski, [Survey article: Bellman function method and sharp inequalities for martingales](#). *Rocky Mountain J. Math.* **43** (2013), no. 6, 1759–1823
- [31] A. Osękowski, [Sharp inequalities for BMO functions](#). *Chinese Ann. Math. Ser. B* **36** (2015), no. 2, 225–236
- [32] A. Osękowski, [Weak- \$L^\infty\$  inequalities for BMO functions](#). *New York J. Math.* **21** (2015), 699–713

- [33] A. Osękowski, *Sharp martingale and semimartingale inequalities*. IMPAN Monogr. Mat. (N. S.) 72, Birkhäuser, Basel, 2012
- [34] A. Osękowski, *A sharp estimate for Muckenhoupt class  $A_\infty$  and BMO*. *Positivity* **23** (2019), no. 3, 711–725
- [35] A. Osękowski, *Embedding BMO into weighted BMO*. *Publ. Mat.* **65** (2021), no. 1, 335–361
- [36] G. Rey, *On the embedding of  $A_1$  into  $A_\infty$* . *Proc. Amer. Math. Soc.* **144** (2016), no. 10, 4455–4470
- [37] A. Reznikov, *Sharp weak type estimates for weights in the class  $A_{p_1, p_2}$* . *Rev. Mat. Iberoam.* **29** (2013), no. 2, 433–478
- [38] V. D. Sedyh, *Singularities of the convex hull of a curve in  $\mathbf{R}^3$* . *Funkcional. Anal. i Priložen.* **11** (1977), no. 1, 81–82 (in Russian). English translation: *Funct. Anal. Appl.* **11** (1977), no. 1, 72–73
- [39] V. D. Sedykh, *Four vertices of a convex space curve*. *Bull. London Math. Soc.* **26** (1994), no. 2, 177–180
- [40] L. Slavin, *The John–Nirenberg constant of  $BMO^p$ ,  $1 \leq p \leq 2$* . 2015, arXiv:1506.04969v1
- [41] L. Slavin and V. Vasyunin, *Sharp results in the integral-form John–Nirenberg inequality*. *Trans. Amer. Math. Soc.* **363** (2011), no. 8, 4135–4169
- [42] L. Slavin and V. Vasyunin, *Sharp  $L^p$  estimates on BMO*. *Indiana Univ. Math. J.* **61** (2012), no. 3, 1051–1110
- [43] L. Slavin and V. Vasyunin, *Cincinnati lectures on Bellman functions*. 2015, arXiv:1508.07668v1
- [44] L. Slavin and P. Zatitskii, *Dimension-free estimates for semigroup BMO and  $A_p$* . *Indiana Univ. Math. J.* **70** (2021), no. 6, 2677–2692
- [45] L. Slavin and P. Zatitskii, *Monotone rearrangement does not increase generalized Campanato norm in VMO*. *J. Funct. Anal.* **288** (2025), no. 7, article no. 110828
- [46] E. M. Stein, *Harmonic analysis: real-variable methods, orthogonality, and oscillatory integrals*. Princeton Math. Ser. 43, Princeton University Press, Princeton, NJ, 1993
- [47] D. Stolyarov, V. Vasyunin, and P. Zatitskiy, *Sharp multiplicative inequalities with BMO I*. *J. Math. Anal. Appl.* **492** (2020), no. 2, article no. 124479
- [48] D. Stolyarov, V. Vasyunin, P. Zatitskiy, and I. Zlotnikov, *Sharp moment estimates for martingales with uniformly bounded square functions*. *Math. Z.* **302** (2022), no. 1, 181–217
- [49] D. Stolyarov and P. Zatitskiy, *Sharp transference principle for BMO and  $A_p$* . *J. Funct. Anal.* **281** (2021), no. 6, article no. 109085
- [50] D. M. Stolyarov, V. Vasyunin, P. Zatitskiy, and I. Zlotnikov, *Distribution of martingales with bounded square functions*. *C. R. Math. Acad. Sci. Paris* **357** (2019), no. 8, 671–675
- [51] D. M. Stolyarov, V. I. Vasyunin, and P. B. Zatitskiy, *Monotonic rearrangements of functions with small mean oscillation*. *Studia Math.* **231** (2015), no. 3, 257–267

- [52] D. M. Stolyarov and P. B. Zatitskiy, [Theory of locally concave functions and its applications to sharp estimates of integral functionals](#). *Adv. Math.* **291** (2016), 228–273
- [53] V. Vasyunin, [The sharp constant in the reverse Hölder inequality for Muckenhoupt weights](#) (in Russian). *Algebra i Analiz* **15** (2003), no. 1, 73–117. [English translation: St. Petersburg Math. J.](#) **15** (2004), no. 1, 49–79.
- [54] V. I. Vasyunin, [Mutual estimates of  \$L^p\$ -norms and the Bellman function](#) (in Russian). *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)* **355** (2008), no. Issledovaniya po Lineinym Operatoram i Teorii Funktsii. 36, 81–138, 237–238. [English translation: J. Math. Sci. \(N. Y.\)](#) **156** (2009), no. 5, 766–798.
- [55] V. I. Vasyunin, [An example of constructing a Bellman function for extremal problems in BMO](#) (in Russian). *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)* **424** (2014), no. Issledovaniya po Lineinym Operatoram i Teorii Funktsii. 42, 33–125. [English translation: J. Math. Sci. \(N. Y.\)](#) **209** (2015), no. 5, 683–742
- [56] V. Vasyunin and L. Slavin, [The John–Nirenberg constant of  \$BMO^p\$ ,  \$p > 2\$](#)  (in Russian). *Algebra i Analiz* **28** (2016), no. 2, 72–96. [English translation: St. Petersburg Math. J.](#) **28** (2017), no. 2, 181–196
- [57] V. Vasyunin and A. Volberg, [Sharp constants in the classical weak form of the John–Nirenberg inequality](#). *Proc. Lond. Math. Soc. (3)* **108** (2014), no. 6, 1417–1434
- [58] V. Vasyunin and A. Volberg, [The Bellman function technique in harmonic analysis](#). Cambridge Stud. Adv. Math. 186, Cambridge University Press, Cambridge, 2020
- [59] V. Vasyunin, P. Zatitskiy, and I. Zlotnikov, [Sharp multiplicative inequalities with BMO II](#). *J. Math. Anal. Appl.* **515** (2022), no. 2, article no. 126430
- [60] V. M. Zakaljukin, [Singularities of convex hulls of smooth manifolds](#). *Funkcional. Anal. i Priložen.* **11** (1977), no. 3, 76–77. [English translation: Funct. Anal. Appl.](#) **11** (1977), no. 3, 225–227
- [61] P. B. Zatitskiy, P. Ivanisvili, and D. M. Stolyarov, [Bellman vs Beurling: sharp estimates of uniform convexity for  \$L^p\$  spaces](#) (in Russian). *Algebra i Analiz* **27** (2015), no. 2, 218–231. [English translation: St. Petersburg Math. J.](#) **27** (2016), no. 2, 333–343
- [62] P. B. Zatitskiy and D. M. Stolyarov, [On locally concave functions on simplest nonconvex domains](#). *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)* **512** (2022), no. Issledovaniya po Lineinym Operatoram i Teorii Funktsii. 50, 40–87. [English translation: J. Math. Sci. \(N. Y.\)](#) **282** (2024), no. 4, 482–510