Abstract

It is by now classical that lattices in higher rank semisimple groups have various rigidity properties. In this work, we add another such rigidity property to the list: uniform stability with respect to the family of unitary operators on finite-dimensional Hilbert spaces equipped with submultiplicative norms. Namely, we show that for (most) high rank lattices Γ, every finite-dimensional unitary "almost-representation" is a small deformation of a (true) unitary representation. This extends a result of Kazhdan (1983) for amenable groups and of Burger–Ozawa–Thom (2013) for $\mathrm{SL}(n, \mathbf{Z})$ (for $n > 2$). Towards this goal, we first build an elaborate cohomological theory capturing the obstruction to such stability, and show that the vanishing of the second cohomology implies uniform stability in this setting. This cohomology can be roughly thought of as an asymptotic variant of bounded cohomology, and sheds light on a question raised by Monod (2006) about a possible connection between vanishing of the second bounded cohomology and Ulam stability.

This memoir is dedicated to the memory of Bob Zimmer in honor of his remarkable achievements and influence on the study of rigidity of lattices in semisimple Lie groups.

Keywords. group stability, bounded cohomology, lattices in Lie groups, amenability

Mathematics Subject Classification (2020): 22D40 (primary); 20J05, 20J06, 22E41 (secondary)

Acknowledgments. The second author acknowledges with gratitude the hospitality and support of MFO Oberwolfach, the Fields Institute (Toronto) and the Institute for Advanced Study (Princeton) where part of this work was carried on. The authors would also like to thank Andrei Rapinchuk for his guidance in the proof of Proposition 7.1.1.

Funding. This work was partially funded by a grant by the European Research Council (ERC) under the European Union's Horizon 2020 (grant agreement No 882751). The results presented here are part of the fourth author's PhD thesis, also supported through this grant.