

Preface

Twenty years of research in cluster theory have established deep links between cluster algebras, surface geometry and representation theory. The focus of this textbook is around the geometric approaches to cluster algebras and to cluster categories: cluster structures can be defined using surface geometry, with certain curves corresponding to cluster variables on one hand and to rigid indecomposable objects of cluster categories on the other hand. Under this, the classical Ptolemy relation or the flip in a triangulation gives rise to the exchange relation or mutation in the associated cluster structures.

In Chapter 1, we start by discussing several examples of cluster algebras before giving their definitions via matrices and quivers. In Chapter 2, we explain how surface triangulations give rise to cluster algebras. We include a discussion of tagged arcs. Furthermore, snake and band graphs are defined as a tool towards an expansion formula for cluster variables. Chapter 3 presents cluster categories, starting from hereditary types. Then their generalized versions are introduced in order to define cluster categories from surface triangulations. In Chapter 4, we discuss two related active research areas: we introduce frieze patterns and explain how surface models can be used to describe categories of modules. In the appendix, we provide background material and hints for the exercises.

These lecture notes grew out of a Nachdiplom lecture course on cluster algebras and cluster categories via surface combinatorics given at the Department of Mathematics at ETH Zürich in Spring 2021, while I was a guest at the FIM (Forschungsinstitut für Mathematik). I am very grateful for this opportunity and I would like to thank Alessandra Iozzi for her hospitality during my stay. I would also like to thank Victor Jaeck whose typed up version of the Nachdiplom lectures formed the basis for these notes and Amandine Favre for the feedback on an earlier version of the notes. I thank the School of Mathematics at the University of Leeds and the Faculty of Mathematics at the Ruhr-Universität Bochum for supporting this project.