

Preface

A matrix nearness problem consists of finding, for an arbitrary matrix A , a nearest member of some given class of matrices, where distance is measured in a matrix norm.

(Nicholas J. Higham, 1989)

This book is about solving matrix nearness problems that are related to eigenvalues or singular values or pseudospectra. These problems arise in great diversity in various fields, be they related to dynamics, as in questions of robust stability and robust control, or related to graphs, as in questions of clustering and ranking. Algorithms for such problems work with matrix perturbations that drive eigenvalues or singular values or Rayleigh quotients to desired locations.

Remarkably, the optimal perturbation matrices are typically of rank one or are projections of rank-1 matrices onto a linear structure, e.g., a prescribed sparsity pattern. In the approach worked out here, these optimal rank-1 perturbations will be determined in a two-level iteration that consists of eigenvalue optimization and root-finding. An eigenvalue optimization problem with equality or inequality constraints on the perturbation size is to be solved via gradient-based rank-1 matrix differential equations. This amounts to numerically driving a rank-1 matrix, which is represented by two vectors, to a stationary point. The root-finding part determines the optimal perturbation size by solving a scalar nonlinear equation. The two algorithmic parts can be nested or can alternate.

A wide variety of matrix nearness problems, as outlined in the introductory Chapter 1, will be tackled by such an approach and its nontrivial extensions. In Chapter 2, we study a basic eigenvalue optimization problem and its numerical solution via rank-1 matrix differential equations, which are norm- and rank-1 constrained gradient systems. In Chapter 3, this approach yields algorithms for computing extremal points and boundary curves of pseudospectra. In Chapter 4, we present algorithms for matrix nearness problems, which in a nested or alternating way combine the rank-1 eigenvalue optimization algorithms of Chapter 2 with a Newton-bisection method. This is illustrated by the problem of computing a nearest unstable complex matrix to a given stable matrix. In Chapter 5, the rank-1 approach is extended to nearness problems for matrices with a prescribed complex-linear or real-linear structure, e.g., real matrices or matrices with a given sparsity pattern or a Toeplitz or Hankel structure. The approach is applied to various structured matrix nearness problems, which include finding the nearest singular structured matrix to a given invertible structured matrix and, for asymptotically stable linear differential equations, finding the largest norm of structured perturbations of the matrix that still ensures a prescribed transient

bound. In Chapter 6, we propose and analyze algorithms for matrix nearness problems that go beyond those of Chapters 4 and 5, among them matrix stabilization and finding a nearest defective real or complex matrix. In Chapter 7, we discuss algorithms for exemplary nearness problems in the area of systems and control, and in Chapter 8, we rephrase clustering and ranking problems from graph theory as structured matrix nearness problems and extend our algorithmic approach to such graph problems.

In Chapters 2 to 8, the inclusion of references to the existing literature is done in Notes at the end of each chapter, and only exceptionally we add references to the running text. References are not numbered but are addressed by the names of authors and the year of publication, for example, Higham (1989) and Lewis and Overton (1996).

MATLAB codes implementing the algorithms presented in this book are freely available for non-commercial, academic purposes and can be obtained upon request from the authors.

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