

Tales of (mathematical) paradise

José Ferreirós

Abstract. Around 100 years ago, Hilbert stated his famous words that “nobody shall expel us” from the Cantorian paradise. The theory of sets had emerged rather suddenly, exactly 150 years ago, with a surprising paper of Cantor that has a peculiar inner story. In this history talk, I presented three vignettes from the early history of set theory. They represent three key moments: 1874, marking the birth of transfinite cardinalities under funny circumstances; 1905, a highly polemical moment when the schism between constructivists and “formalists” emerged; and 1925, the high time of the foundational debate between critics (Weyl, Brouwer, Skolem) and partisans of the higher infinite. I will not follow the obvious chronology; the narrative will be like those movies that include flashbacks.

“Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können.” (Hilbert, in Münster on 4 June 1925, during a meeting in memory of Weierstraß.)

Almost a century ago, David Hilbert was saying his famous words, “nobody shall expel us” from the Cantorian paradise. Some of his contemporaries believed that the purported “paradise” was completely artificial. We shall revisit some facets of this debate in this history talk. I will be presenting three scenes or vignettes from the early history of set theory, covering a period of 50 years. They represent three key moments: 1874, marking the birth of transfinite cardinalities under peculiar circumstances; 1905, a highly polemical moment when the divide between constructivists and so-called “formalists” emerged; and 1925, the high time of the foundational debate between critics (Weyl, Brouwer, Skolem) and partisans of the higher infinite.

I intend to provide you with some interesting personal details concerning the real people involved in the story (mathematics is a human endeavor!), and also some details that have to do with methodological subtleties of this story. In this way, we will come to understand some reasons why the problems of set theory ended up splitting the mathematical community into parties that disagreed.

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I hope you will excuse me if I do not follow the obvious chronology. Let me begin a century ago, with Hilbert and his famous conference “On the Infinite.” Then we will move back in time to 1874.

1. 1925: Hilbert in paradise?

In 1925, the foundational debate between intuitionism and formalism was at its peak. This began to get heated in 1921 with Weyl’s intervention; reactions to intuitionism started to gain momentum, with a peak around 1925–1926.¹

In 1924 and 1925, there was quite a lot of activity, with the brilliant intuitionist L. E. J. Brouwer very actively publishing papers,² Hilbert delivering his speech on the infinite, Weyl publishing a couple of papers, plus several other lesser figures – Fraenkel, Gönseth, Kolmogorov, Lietzmann, etc.

Historians have remarked that the tone of the debate was very political and allegorical from the beginning: Brouwer was called “the revolution,” Weyl and Brouwer a “repeated attempt of a putsch,” their views compared with Kronecker’s (vs. the good Cantor, who is the point of comparison for Hilbert); and then Hilbert refusing to abandon the Paradise (a rebellious Adam!), “in the beginning was the sign,” and so on.³

But besides the personal and political undertones, there were clear foundational logical topics, concerning the justification for actual infinity, the status of pure existence principles like AC, or the need to restrict methods to constructive procedures. This was the context for Hilbert’s famous talk “On the Infinite” in 1925.

That is well remembered as the paper in which he included the famous words: “From the Paradise which Cantor created for us, no one shall expel us.” Once again, as he had for many years, he stood forcefully on the side of set theory and so-called “classical” mathematics. The work is also remembered as a detailed and subtle presentation of the Hilbert program for metamathematics, whose goal was to provide a consistency proof for the core mathematical theories (Peano arithmetic, real analysis, set theory).

¹See Hesselning (2003).

²Brouwer in the *Math. Ann.* (1925), *Crelle* (1924), and *Jb.DMV* (1925). Weyl in *Math. Zeitschr.* (1924) and a popular presentation in the philosophical journal *Symposion*.

³The whole thing would explode in 1928, with the “battle of frogs and mice,” only partially due to foundational reasons – perhaps more due to the Bologna ICM in September 1928. Brouwer promoted a boycott, and there was an increasing “incompatibility of our views on fundamental matters” (D. Rowe and H. Mehrtens have written on this topic; see also van Dalen 1990).

This is not the place to go into the details of the Hilbert program, but I would like to revisit the main philosophical lessons that he tried to establish. These are somewhat ambivalent and thus surprising.

Hilbert begins with a lengthy discussion of the infinite in natural science. He argues in detail that neither in the direction of the infinitely small nor in the infinitely large does one find infinity. So reality is finite, the infinite is not real in the sense of *wirklich*, actual in the physical world. “Yet it could very well be the case that the infinite has a well-justified place *in our thinking* and plays the role of an indispensable notion.”

But Hilbert seems to be of two minds here: on the one hand, he insists on the whole Cantorian universe, for set theory gives us “the deepest insight into the nature of the infinite”; this is the Romantic side, full of enthusiasm for the transfinite numbers, for the richness of mathematics. Cantor’s “theory of *transfinite numbers* appears to me to be the most admirable flower of the mathematical intellect and in general one of the highest achievements of purely rational human activity.” But there is also a critical side, and he ends up denying the infinite a substantive role in our thinking.

In the middle of the talk, we find the famous passage discussing “a completely satisfactory way of avoiding the paradoxes without betraying our science.” The right attitude and viewpoints are these:

- (1) We will carefully investigate and promote fruitful definitions [*Begriffsbildungen*] and deductive methods, wherever there is any hope of salvage. We will nurse them, strengthen them, and make them useful. No one shall drive us out of the paradise which Cantor created for us.
- (2) We must establish throughout mathematics the same certitude for our deductions as exists in ordinary elementary number theory, which no one doubts and where contradictions and paradoxes arise only through our own carelessness.

Obviously, these goals can be attained only after we have fully elucidated *the nature of the infinite*.

But surprisingly, the Göttingen mathematician ends up denying the infinite a substantive role and relegating it to the status of a regulative idea in a Kantian sense:

The final result then is: nowhere is the infinite realized; it is neither present in Nature nor admissible as a foundation in our rational thinking – a remarkable harmony between being and thought.⁴ ... The right to operate with the infinite

⁴This idea runs contrary to Cantor’s philosophical position regarding the infinite, and we can be certain that he would have been unhappy with Hilbert’s views.

can be secured only by means of the finite [i.e., justifying the procedure by proving its consistency].

Here, we find the critical side of Hilbert's, far from the above-mentioned Romanticism, where he is trying to accommodate the thought that it is all "ideal elements" and a *manner of speech* (*façon de parler*) which boils down to finite procedures. Thus, he ends by saying:

The role that remains to the infinite is, rather, merely that of an Idea – if, in accordance with Kant's words, we understand by an Idea a concept of reason that transcends all experience and through which the concrete is completed so as to form a totality – an idea, moreover, in which we may have unhesitating confidence within the framework furnished by my theory. (Hilbert 1925, p. 392)

This last is merely a hope, which was later contradicted by Gödel's arch-famous results.

We see that Hilbert was to some extent ambivalent, of two hearts concerning the higher infinite. He stands forcefully on the side of set theory, but his critical analysis of the foundational situation inclines him toward skepticism, or at least to a moderate standpoint. That was also the situation for many other mathematicians at the time.

Indeed, in the 1920s, there were important attempts to ground the Zermelo–Fraenkel axiom system ZFC, but also to restrict it, to the despair of Zermelo. Few people were as optimistic as Hilbert; many expressed an attitude of *neutrality* toward the actual infinite – this applies even to great experts like Sierpiński and Luzin – and some leading mathematicians rejected the Cantorian paradise.

Let me give just two examples. Émile Borel had been important by introducing Cantor's work in France, but after the discussion on the AC and the well-ordering theorem, in 1905, he turned into a critic of set theory (even rejecting the notion of $\aleph_1$). And he was highly influential. We will come back to this in the last section.

Another example is the leading mathematician in Spain and Argentina at the time, Julio Rey Pastor (1888–1962). His book *La matemática superior* was written in 1916 under the influence of a stay at Göttingen shortly before. But in the second edition, in 1951, Rey Pastor distances himself from his "youthful enthusiasm" for set theory. In 1916, he had explained that mathematics would be defined in the future as "the *Science of Sets*." In the later edition, he changed the first chapter of the book and wrote:

At the beginning of the 20th century, ephemeral golden age of Cantorism, one considered the transfinite Arithmetic and Geometry of Cantor as the indispensable basis of Analysis; the new treatises devoted a long initial chapter to them. The successes of the French school (Borel, Baire, Lebesgue, Fréchet)

confirmed this prestige, while the criticism of Borel and Poincaré moderated the excessive enthusiasm, making clear that the whole of classical Analysis can be rigorously built without the actual infinite but with some Cantorian concepts that are indispensable to promote the theory of real functions... (accumulation points, neighborhoods,...). The rise of the unexpected paradoxes, early in the century, cooled down the enthusiasm of which this author was infected, giving the transfinite undue prominence in these conferences. (Rey Pastor 1951, pp. 20–21, my translation)

According to him, mathematics in the 19th century, while going from the concrete to the abstract, had never lost sight of reality and the natural sciences. Meanwhile, it is “the characteristic of our times: arbitrary abstract constructions, arbitrary postulates and arbitrary functions; a sum of arbitrariness that horrified Poincaré” (Rey Pastor 1951, p. 7).

Similar second thoughts about set theory can be found in many relevant people: Poincaré, Lebesgue, Baire, Weyl, Skolem, van der Waerden, etc. As I have explained elsewhere, in the interwar period, an “atmosphere of insecurity” reigned. But let us now move back in time and consider the beginnings.

2. 1874: Birth in sin

Set theory was born in 1874 with the publication of Cantor’s paper “On a property of the collection of all real algebraic numbers.”⁵ It was born in sin, as we shall see.

The contents of this elegant 1874 paper took form in the correspondence between Cantor and Dedekind, November–December 1873. The work presents two theorems, neither of them trivial at the time: a proof of the denumerability of the algebraic numbers, and a proof of the non-denumerability of $\mathbb{R}$. Those results provide an “essential distinction” between two kinds of infinite sets and a proof of the existence of transcendental (non-algebraic) numbers.

Here are some inconvenient facts we know:⁶

- (1) The first theorem was proven by Dedekind in a letter on 30 November.
- (2) The title highlights the first theorem and Dedekind’s main working field, i.e., algebraic number theory.

⁵Cantor (1874). This assertion of course is somewhat exaggerated. Set theory was already beginning some years before (see Ferreirós 2024). But in this 1874 publication, it was shown, for the first time, that infinite sets with different “sizes” exist. In this sense, it was the beginning of *transfinite* set theory.

⁶From the published correspondence, Cantor and Dedekind (1937), also available in English.

- (3) Notes are preserved in which Dedekind expresses surprise at how Cantor used his letters (30 November and 8 December) “almost literally” in print, *Crelle’s Journal*, vol. 77.
- (4) The idea of the second proof was certainly Cantor’s, but Dedekind simplified it very much in one of his letters (8 December) and modified it somewhat.⁷
- (5) All was somehow related to the “Berlin circumstances” (which means, to Weierstraß and Kronecker), but Dedekind never received more details – they “never discussed the paper” later on.⁸

Indeed, Weierstraß had a great interest in the first theorem (algebraic numbers ordered into a simple countable sequence) and encouraged Cantor to publish.

On pondering these facts, one is led to wonder – a text written by whom, and who should be credited with the theorems? I suggest it should have been a joint paper. And then, there is the matter of the title! Dedekind had just begun to make his mark (1871) in algebraic number theory, indeed offering a kind of redefinition of higher number theory. The combination of the use of his letters *verbatim*, and the insertion of that title, must have been mortifying.

Instead of presenting my own evaluation of the situation, let me just cite what other experts think. S. Müller-Stach, who has written about Dedekind’s life and work (2024), speaks of Cantor’s “unthoughtful publication” of some common ideas “without mentioning Dedekind, thus violating scientific standards boldly.” He remarks that a letter from Dedekind’s nephew to Aunt Julie Dedekind from 1887 proves that this was a well-known issue in the entire Dedekind family. This may sound like a mere anecdote, but it is indeed a very telling one.

Oliver Deiser, who is much more favorable to Cantor,⁹ writes: “As a compromise, one might in the future call the result of countability of the algebraic numbers, Theorem of Cantor–Dedekind. The rest of the complex may remain with Cantor.” Cantor reacted to Dedekind’s proof of denumerability with the comment that his method was similar to how he proved the denumerability of tuples of integers. That is the basis for Deiser’s claim that the theorem is due to both. However, Cantor’s method cannot be extended to the case of the algebraic numbers unless one employs the key concept of the “height” of a polynomial, due to Dedekind (see Ferreirós 1993, p. 358).

⁷It seems this version went verbatim into the paper, except for five words (hence the “almost”). In another letter, Cantor acknowledged that Dedekind’s “comments and expressions” were “so valuable to me” that he “thought it appropriate to adopt” them (!).

⁸The strange circumstances of the 1874 paper may have involved a certain quarrel between Kronecker (and Kummer) and Dedekind – a typical priority issue, both being experts in the theory of algebraic numbers. Compare Dedekind’s *Werke*, vol. 3, 481.

⁹See Deiser’s website <https://www.aleph1.info/?call=Puc&permalink=cd1> (and search for Die Veröffentlichung von Cantor 1874).

In any event, the personalities and attitudes of both mathematicians were quite different and apparently sometimes incompatible.

But there is a *second reason* to talk about sinful birth. This is because of the deceitful clarity and perspicuity involved in the definitions of the real numbers that Cantor and Dedekind put forward (1872). They seemed to provide a perfectly precise characterization of the real-number continuum, but neither of them was fully aware of the import of their actual presuppositions. What I mean is the (implicit) powerset principle and the vague notion of arbitrary subsets.

The 1872 papers rely, respectively, on Dedekind cuts on $\mathbb{Q}$ and on fundamental sequences of rational numbers (a.k.a. Cauchy sequences). In order to obtain the complete system of the real numbers, this means relying on *arbitrary* sequences of rationals with the Cauchy property (Cantor), or equivalently, on arbitrary cuts on $\mathbb{Q}$ (Dedekind). They assume given a *totality of all* such arbitrary infinitary constructs, considered to be extensionally definite (not merely intensionally defined).¹⁰

In the technical terminology of set theory, Dedekind and Cantor were implicitly (and unclearly) adopting the *quasi-combinatorial* conception of sets. That they were so doing was only gradually clarified, especially through the criticism leveled against their work by constructivist mathematicians (Kronecker, Poincaré, Borel, Baire, Brouwer, Weyl, Skolem, and others). One can see how the critics did a great service to set-theoretic mathematics by helping to clarify its foundations.

Let me put it differently, to be more clear: it was explicit already (as of 1872) that these definitions relied on *actually infinite* sets, but it was left obscure the way in which they presupposed *arbitrary* or “*random*” sets (not merely “*lawful*” sets, given by a definite law) – and moreover an alleged totality of all such.

This situation was acknowledged years later, e.g., by Hilbert [11], when he mentioned “the doubts which have been raised against the existence of the totality of all real numbers” based on the notion that “by the set of real numbers” we have “to imagine, say, the totality of all possible laws according to which the elements of a fundamental sequence can proceed.” The situation is even worse, since we assume that there are sequences that do not accord with *any* law.

¹⁰This distinction is due to Weyl; see the detailed explanation in Crosilla and Linnebo (2024). The idea is this: a condition is *intensionally* definite if, given some object, either the condition applies to that object or it does not; on the other hand, a condition or collection can be definite in the sense that the totality of all its instances or members has been circumscribed – this is *extensional* definiteness.

3. 1904–1905: Open polemics

In the decade of 1900, there were big polemical debates concerning the strange or surprising features of set theory: its strongly non-constructive orientation, its acceptance of “arbitrary” sets, and AC (the axiom of choice). The polemics was mostly about the question which methodology was admissible, especially in the context of the study of infinite sets – hence also about the acceptance or rejection of some fundamental axioms.

Leaving behind early ideas about logicism, this was mostly a debate between modern “formalists” (those in favor of the free methods and fruitful conceptual developments typical of set theory) and critical constructivists (critics of the “arbitrariness” of set theory, partisans of more restrictive methods).

The context included several things. First, from 1899 to 1903, the set-theoretic paradoxes were “discussed in many meetings” (Bernstein), and perhaps more widely after the publication of books by Russell and Frege in 1903.¹¹ But second, and even more important, there was a heated debate about the Continuum Problem at the 1904 ICM. This was Hilbert’s [10] problem no. 1: Cantor’s continuum hypothesis, CH, and the well-ordering theorem.

At the 1904 ICM in Heidelberg, Gyula König proved that the continuum is no aleph, in the presence of both Cantor and Hilbert. This caused a sensation; it is reported that the other sessions of the congress were rescheduled so that all participants could attend König’s presentation. We have a moving description of Cantor’s reaction:

Then Cantor took the word in a state of deep excitement. He expressed his gratefulness to God for having allowed him to live and see this refutation of his errors. The newspapers informed of König’s talk. The Grand Duke of Baden himself asked to be informed of these sensational events through Felix Klein. (Kowalewski 1950, pp. 198–203)

Unfortunately, one cannot always trust reminiscences.¹² A real eyewitness of the events gave a totally different account: “It is characteristic of Cantor that, from the beginning, he considered König’s result as invalid, despite his exact demonstrations.

¹¹As Cantor insisted (letters to Hilbert and Dedekind, 1899), there were reasons against the notions of a “collection of all” ordinals, of “all” cardinals or alephs, and even an actual infinity of “all” sets. All of this, made more complex due to many other “logical” paradoxes (Russell 1903, Richard 1905, etc.), later distinguished as semantic paradoxes.

¹²According to the Acts of the ICM, after König’s talk *Zum Kontinuum-Problem* Cantor, Hilbert and Schoenflies took the word. Kowalewski was not present at the 1904 ICM.

[He rejoiced in saying that he did not mistrust the König, but only His Minister].” (Schoenflies 1922, p. 100)

In reaction to König’s proof, a group of mathematicians including Cantor, Hilbert and Zermelo joined to discuss the situation and clarify it. If true, König’s result would have destroyed the edifice of Cantor’s transfinite set theory. Later in 1904, contributions due to Hausdorff and Zermelo again transformed the situation. Hausdorff clarified the issue with König’s theorem, showing that his result had to be restricted: the fault was not due to König himself (all his original arguments were valid) but to a theorem which had been incorrectly formulated in Bernstein’s dissertation;¹³ ironically, Bernstein was a doctoral student of Cantor and Hilbert. Then, in September 1904, Zermelo sent his proof of the well-ordering theorem based on the axiom of choice, AC, in the form of a letter. It was immediately published in the *Mathematische Annalen*, whose editors were Klein and Hilbert.

Notice that the well-ordering theorem implies that every set has an aleph as its cardinal number. It is a very significant strengthening of the Cantorian edifice. But Zermelo’s argument raised many doubts. Immediately, there came reactions to be published in the *Math. Annalen* (see vols. 60 and 61). A huge debate followed, a surprisingly intense series of arguments and counter-arguments, with authors from all the main European countries: France, Germany, Britain, Italy, with the “five letters” exchanged in France,¹⁴ plus contributions due to Peano, Poincaré, Hardy, Jourdain, Russell...

There was confusion because of numerous sources of doubt: not only the new axiom AC, which was very explicitly non-constructive. Even those willing to accept it believed that there could be other problems with the proof, e.g., that it might depend on the unacceptable “set” of all ordinals. This was the case of Bernstein. Zermelo clarified the situation in 1908, with a new proof of the theorem, an explicit defense of the axiom, and the presentation of a whole axiom system for set theory.

It is instructive to consider that Hilbert, in his famous talk on Mathematical Problems [10], was asking for an *effective* well-ordering of the reals, a constructive solution to the problem of well-ordering $\mathbb{R}$:

The question now arises whether the totality of all [real] numbers may not be arranged in another manner so that every subset may have a first element, i.e., whether the continuum cannot be considered as a well-ordered set—a

¹³A result which is only valid for alephs with a limit ordinal subindex, not for successor ordinals – hence, valid for $\aleph_\omega$ but not for $\aleph_1, \aleph_2, \dots$

¹⁴Published in the *Bull. Société Mathématique de France*; see Borel et al. (1905). The *Math. Annalen* vol. 60(2) featured four papers discussing Zermelo’s short piece (counting up to seven with vol. 61).

question which Cantor thinks must be answered in the affirmative. It appears to me most desirable to obtain a direct proof of this remarkable statement of Cantor's, say, by actually giving an arrangement of numbers such that in every subsystem a first number can be pointed out.

Nobody has found such a constructive solution, and it is consistent with ZFC that there is none. What Zermelo provided, instead, was a totally non-constructive solution of the problem.

Many other people were also requesting an effective, constructive solution. For Émile Borel, the result of Zermelo was merely *conditional*: if you can define a choice function (on the powerset of S), *then* you can well-order the set (S); otherwise, your hands are empty. Borel continued to debate the problem for years, and he was highly influential. In his papers and talks, he came to the point of rejecting the notion of $\aleph_1$ as ill-defined.¹⁵

But notice that the axiom AC is perfectly natural, provided that we understand set theory to deal with *arbitrary infinite sets*. It applies to “every set for which the totality of subsets, etc. is meaningful [*einen Sinn hat*],” as Zermelo said. However, does the idea of a completely arbitrary function make actual sense? That is the problem which led to a split among mathematicians.

The debate would continue for many years to come, even if Hilbert's side was winning in practice. To give one further example, let me quote a paper by O. Perron in 1911:

Today there is a branch of mathematics, in which the viewpoints are very divided; some take for correct, what the others reject. It is the so-called theory of sets, in which the security of mathematical deduction seems to have been completely lost.

The worst contrapositions arise in the question of the well-ordering of the continuum. Where one half regards the proof as absolutely free of difficulties,... the other half declares it all false. (Perron 1911)

One can give further examples of critics of set theory among relevant mathematicians in all generations from 1904 until 2024. What will be the future of set theory? Despite enormous technical progress in the field during the last century, some reasons for skepticism have been present, most obviously the Continuum Problem (whose “resolution” or lack thereof may be construed as a reason for distrust).

¹⁵This situation was even interpreted as a *refutation* of the AC and the well-ordering theorem – assuming impossible an explicit well-ordering of $\mathbb{R}$.

Was the Cantorian paradise, after all, just an artificial paradise?¹⁶

References

- [1] E. Borel et al., Cinq lettres sur la theorie des ensembles. In his 1950, *Leçons sur la théorie des fonctions*, 4th edn. Gauthier-Villars, Paris, 1905; 1st edn., 1898 (English translation as appendix to G. H. Moore, *Zermelo's Axiom of Choice*. Springer, Berlin, 1982)
- [2] G. Cantor, [Ueber eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen](#). *J. Reine Angew. Math.* **1874** (1874), no. 77, 258–262 (Also in his *Abhandlungen*, 115–118. English translation in Ewald 1996, Vol. 2) MR [1579605](#)
- [3] G. Cantor and R. Dedekind, In E. Noether and J. Cavaillès (eds.), *Briefwechsel Cantor–Dedekind*. Hermann, Paris, 1937, 61 pp. (English translation in Ewald 1996, Vol. 2) Zbl [0017.38502](#)
- [4] L. Crosilla and Ø. Linnebo, [Definiteness in early set theory](#). *J. Philos. Math.* **1** (2024), 43–62
- [5] O. Deiser, Die Veröffentlichung von Cantor 1874 (search for the Anhänge). In *Der Briefwechsel zwischen Cantor und Dedekind* (online). 2020, <https://www.aleph1.info/?call=Puc&permalink=cd1> visited on 8 May 2026
- [6] W. Ewald, *From Kant to Hilbert. A source book in the foundations of mathematics*, Vol. II. Oxford University Press, Oxford, 1996 Zbl [0859.01002](#) MR [1465678](#)
- [7] J. Ferreirós, [On the relations between Georg Cantor and Richard Dedekind](#). *Historia Math.* **20** (1993), no. 4, 343–363 Zbl [0792.01031](#) MR [1248470](#)
- [8] J. Ferreirós, The early development of set theory. In E. N. Zalta and U. Nodelman (eds.), *The Stanford Encyclopedia of Philosophy*, Winter 2024 edn., <https://plato.stanford.edu/archives/win2024/entries/settheory-early/> visited on 8 May 2026
- [9] D. E. Hesselning, *Gnomes in the Fog. The reception of Brouwer's intuitionism in the 1920s*. Sci. Networks Hist. Stud. 28, Birkhäuser, Basel, 2003, 448 pp. Zbl [1037.01008](#) MR [2023449](#)
- [10] D. Hilbert, Mathematische probleme. In his *Werke (1932/35)* **3** (1900), 290–339 (English translation in *Bull. Amer. Math. Soc.* **8** (1902), 437–479)
- [11] D. Hilbert, Über den Zahlbegriff. *Jahresber. Dtsch. Math.-Ver.* **8** (1900), no. 1, 180–194 (English translation in Ewald 1996, Vol. 2) JFM [31.0165.02](#)
- [12] D. Hilbert, [Über das Unendliche](#). *Math. Ann.* **95** (1926), no. 1, 161–190 (English translation in J. van Heijenoort, *From Frege to Gödel*, pp. 367–392, Harvard University Press, 1967) JFM [51.0044.02](#) MR [1512272](#)

¹⁶As Dieudonné wrote; see P. Dugac, *Richard Dedekind et les fondements des mathématiques* (Paris, Vrin, 1976), p. 11.

- [13] G. Kowalewski, *Bestand und Wandel*. R. Oldenbourg, München, 1950, 309 pp.
Zbl [0037.00203](#)
- [14] S. Müller-Stach, *Richard Dedekind. What are and what should the numbers be? Continuity and irrational numbers*. Class. Texts Sci., Springer, Berlin, 2024, 236 pp.
Zbl [1560.01051](#) MR [4881998](#)
- [15] O. Perron, Über Wahrheit und Irrtum in der Mathematik. *Jahresber. Dtsch. Math.-Ver.* **20** (1911), 196–211 JFM [42.0073.03](#)
- [16] J. Rey Pastor, *La matemática superior*. Iberoamericana, Buenos Aires, 1951; 1st edn., 1916
- [17] A. Schoenflies, Zur Erinnerung an Georg Cantor. *Jahresber. Dtsch. Math.-Ver.* **31** (1922), 97–106 JFM [48.0019.02](#)
- [18] D. van Dalen, The war of the frogs and the mice, or the crisis of the *Mathematische Annalen. Math. Intelligencer* **12** (1990), no. 4, 17–31 Zbl [0723.01006](#) MR [1076531](#)
- [19] E. Zermelo, Beweis, daß jede Menge wohlgeordnet werden kann. *Math. Ann.* **59** (1904), no. 4, 514–516 (in Z. Werke, Vol. 1 (2010), 80–119. English translation in van Heijenoort, op. cit. (1967). See also the 1908 papers in *Werke*, Vol. 1, 120–230) JFM [35.0088.03](#) MR [1511281](#)

José Ferreirós

Department of Logic and Philosophy of Science, Universidad de Sevilla, C/ Camilo J. Cela s/n, 41018 Sevilla, Spain; josef@us.es