

The work of Katrin Tent

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This volume developed out of the conference *Model Theory and Groups* (25–29 September 2023, University of Münster) to celebrate the 60th birthday of Katrin Tent. Katrin has made, and continues to make, highly original contributions to group theory, to model theory, and to the terrain they share. She has also led the creation of Münster into one of the leading centres for model theory worldwide, and has had huge influence on younger model theorists, and thereby on the direction of the subject, through hosting top postdocs and very successful PhD students. Personally, my collaborations with Katrin over the last 30 or so years have greatly enriched my work.

Katrin, remarkably, has two PhDs, neither really reflecting her subsequent work. The first, completed at the University of Kiel in 1988, was in linguistics. After obtaining a Diplom in Mathematics at Kiel in 1989, she moved to the University of Notre Dame, where she completed a PhD in model theory in 1994 with Steve Buechler as advisor. This second PhD, in geometric stability theory, had title ‘Classifying totally categorical groups (and others)’.

After this second PhD, Katrin held a two-year postdoctoral position at the Hebrew University in Jerusalem, and then moved to the University of Würzburg in 1996, where she worked until 2004, obtaining her Habilitation in 2001 with a thesis ‘Model theory of groups and BN-pairs’. She then briefly moved to her first permanent job at the University of Birmingham, before returning to Germany, to the University of Bielefeld. She moved to Münster in 2008 as Professor of Mathematics and Mathematical Logic, and has remained there since then.

Katrin’s contributions are rich and varied, but there is a distinctive theme which she has honed and led, linking model theory to Tits buildings, BN pairs, and related group-theoretic constructions, especially of simple groups, and, most strikingly, of sharply multiply transitive groups. Much of her earlier work was around groups of finite Morley rank, in particular the still-open algebraicity conjecture, otherwise known as the Cherlin–Zilber conjecture, that every infinite simple group G of finite Morley rank is a simple algebraic group over an algebraically closed field. Katrin’s vision here has been a geometric proof, recovering algebraic group structure from geometric information, in the spirit of Tits. In [4], the authors show that if G has a spherical BN-pair of Tits rank at least 2 associated with a ‘Moufang building’, then the algebraicity conjecture holds for G . They first show that the problem reduces to the ‘rank 2 residues’, incidence structures of points and lines which will be ‘generalised polygons’ (a generalisation of projective planes, crucial to the structure of Tits buildings) which inherit the strong ‘Moufang’ symmetry condition. Moufang generalised polygons exist only for $n \in \{3, 4, 6, 8\}$, and are classified in a book of Tits

and Weiss. The bulk of the paper applies this classification to prove the theorem. The result is important, for example, for recognition theorems of Lie rank 2 groups in [1].

Katrin has frequently revisited questions around generalised polygons. A key step in the first-generation proof of the classification of finite simple groups was the 1973 Fong–Seitz classification of finite groups with an irreducible spherical BN-pair of rank 2 (with an extra condition). In two papers [10, 12] in *Advances*, the first joint with van Maldeghem, Katrin gives a new and much shorter proof of the Fong–Seitz theorem, which, unlike the original proof, applies also without the finiteness assumption. It follows from the theorem that such groups correspond to the Moufang generalised polygons classified by Tits and Weiss (see above). Then, in the remarkable paper [9], she uses a Hrushovski-style construction to exhibit, for each $n \geq 3$, $2^{\aleph_0}$ non-isomorphic strongly minimal countable generalised n -gons with a strong symmetry condition (the collineation group is transitive on simple closed paths of length $2n + 2$ starting with a point). Despite this strong symmetry, none of these generalised n -gons interprets an infinite group, making them highly non-classical.

The hardest problems around Tits buildings and BN-pairs concern those of rank at most 2, just as projective planes are much harder to describe than projective spaces of higher rank. Rank 2 leads to generalised n -gons, and rank 1 leads to 2-transitive groups. In particular, *sharply* 2-transitive groups (where in addition each 2-point stabiliser is trivial) have been a theme of Katrin’s work since her 2000 paper [8], where she classifies sharply 2-transitive and sharply 3-transitive permutation groups definable in an o-minimal structure. *Finite* sharply 2-transitive groups (G, X) have been known since work of Zassenhaus in the 1930s to be *split* and to arise from *near-fields*; that is, such G has a regular normal subgroup N consisting of the identity and fixed-point-free elements, and G is a semidirect product $G = N \rtimes G_x$, where G_x is the stabiliser of $x \in X$. The question of whether every *infinite* sharply 2-transitive group has an abelian normal subgroup was well known and long open, only answered by Rips, Segev and Tent in the spectacular paper [6]. There it is shown that *any* group G is contained in a group G^* which has no abelian normal subgroup and acts sharply 2-transitively on a set X in such a way that no involution has a fixed point; the last condition is referred to as having *permutational characteristic 2*. This work has opened up a whole new field, and Katrin has co-authored a number of related papers on this theme, for example [15] giving an alternative construction, [11] giving sharply 3-transitive groups whose point stabiliser has no non-trivial abelian normal subgroup, [2] exhibiting *simple* sharply 2-transitive groups, along with several papers paying attention to permutational characteristic.

Katrin also has impressive work on infinite simple groups, both through interesting constructions, and through proofs of simplicity. With her former PhD student Zaniar Ghadernezhad [3], she exhibited a striking phenomenon: a non-algebraic simple group with a (non-split) BN-pair of rank 2. The group is basically the automorphism group of one of the Hrushovski-style generalised polygons mentioned

above, with simplicity proved via a result of Lascar. The latter ‘Lascar technique’ was applied earlier in a paper by Katrin and myself [5] to show that automorphism groups of transitive and non-trivial Fraïssé limits of ‘free amalgamation classes’ are simple. This was developed into a readily-usable technique by Katrin and Martin Ziegler in [14], which isolated the key and now widely-used concept of ‘stationary independence’.

Katrin has many other papers of real importance that I won’t cover here: for example around geometric group theory and the model theory of free groups; on connections of group theory to descriptive set theory; and work with Segal [7] on, for a Chevalley group G and domain R , the problem of defining R in the group $G(R)$ of R -rational points. Her beautiful 2012 book with Ziegler *A course in model theory* [13] is surely the main postgraduate-level model theory textbook.

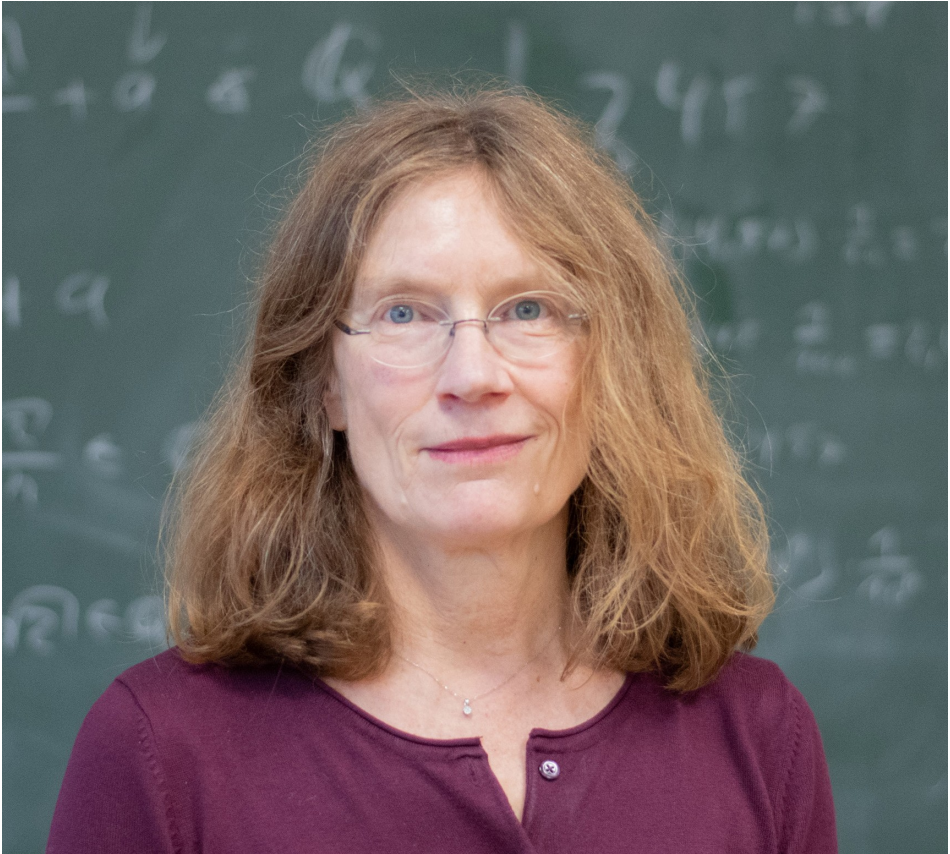
Katrin and Münster. Katrin has built Münster into one of the leading centres for model theory, with two more recent appointments (Martin Hils and Franziska Jahnke), whilst many top model theorists and group theorists have passed through Münster as her postdocs or PhD students. She has supported our subject by frequently co-organising Oberwolfach workshops, as well as programmes at the HIM in Bonn and workshops elsewhere, for example in Banff, reaching out to core communities in both group theory and model theory. I visited her already several times in Würzburg and Bielefeld, and have enjoyed many research visits to her in Münster. I have also twice heard her sing in the Münster opera, which she somehow manages to combine with mathematics and her many family engagements.

We are very lucky to have Katrin in our field.

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