Abstract

We define a natural notion of higher-order stability and show that subsets of $\mathbb{F}_p^n$ that are tame in this sense can be approximately described by a union of low-complexity quadratic varieties, up to linear error. This generalizes the arithmetic regularity lemma for stable subsets of $\mathbb{F}_p^n$, proved in earlier work of the authors, to the realm of higher-order Fourier analysis.

This result is strictly stronger than the structure theorem for sets of bounded VC_2-dimension, first proved by the authors in earlier versions of this memoir and now available as a separate article. Taken together, these results provide group-theoretic analogs of results obtained by the authors for 3-uniform hypergraphs.

Keywords. model-theoretic stability, NFOP_2, functional order property, arithmetic regularity lemmas, quadratic Fourier analysis

Mathematics Subject Classification (2020): 03C45 (primary); 11B30 (secondary)

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