Abstract

This memoir introduces the trivial fiber topology on schemes. For one-dimensional base schemes, we use it to describe fibrant replacements in the stable motivic homotopy category and motivic infinite loop spaces. We also extend the Garkusha–Panin and Voevodsky strict $\mathbb{A}^1$-invariance theorems to one-dimensional base schemes. The trivial fiber topology plays a central role in the proof of refined localization results for motivic homotopy categories. Moreover, we extend Morel's $\mathbb{A}^1$-connectivity theorem on Nisnevich sheaves of stable motivic homotopy groups. These results open new vistas for computations of motivic invariants over deeper base schemes of arithmetic interest.

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Keywords. motivic homotopy theory, framed correspondences, trivial fiber topology, strict $\mathbb{A}^1$-invariance, $\mathbb{A}^1$-connectivity

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