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Degenerate complex Monge-Ampère equations.
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This comprehensive monograph gives an excellent exposition of pluripotential theory in Euclidean space and on compact complex manifolds, with emphasis on the solutions to complex Monge-Ampère equations and on important applications to complex geometry. It is organized in four parts and sixteen chapters.

The first part is a self-contained presentation of pluripotential theory on domains in $\mathbb{C}^n$. Potential theory in several complex variables, or briefly pluripotential theory, deals with the study of plurisubharmonic functions and positive closed currents. The theory provides powerful tools that led to significant advances in complex analysis, geometry and dynamics. The book starts by developing the basic properties of harmonic, subharmonic, plurisubharmonic functions in Chapter 1, and of positive closed currents in Chapter 2. A central role in pluripotential theory is played by the complex Monge-Ampère operator which associates to a suitable plurisubharmonic function u the positive measure $(dd^c u)^n$. It provides a generalization to higher dimensions of the Laplacian of a subharmonic function in $\mathbb{C}$. Chapters 3–5 give a nice account of the Bedford-Taylor theory on defining the complex Monge-Ampère operator for locally bounded plurisubharmonic functions, the Monge-Ampère capacity and its applications, and the solution to various Dirichlet problems for the Monge-Ampère equation. In Chapter 6 the authors show that the general method of viscosity can be used to solve Monge-Ampère equations on domains in $\mathbb{C}^n$, by developing the corresponding notions and techniques in the complex case.

In the second part the authors extend the notions and results of pluripotential theory from the local setting to that of compact Kähler manifolds. In this case, the plurisubharmonic functions, which by the maximum principle must be constant, are replaced by the quasiplurisubharmonic (qpsh) ones. Important topics from complex geometry are reviewed without proof in Chapter 7, while the class of qpsh functions and the corresponding envelopes and capacities are discussed in Chapters 8–9. Chapter 10 deals with the definition and properties of the complex Monge-Ampère operator acting on suitable classes of unbounded qpsh functions. Of special importance is the class $\mathcal{E}$ on which the complex Monge-Ampère operator can be defined by taking advantage of the compact setting, as well as several subclasses of qpsh functions of finite energy. A new phenomenon is that the class $\mathcal{E}$ contains unbounded qpsh functions for which the complex Monge-Ampère operator cannot be locally defined by the methods of Bedford and Taylor, or, more generally, Błocki and Cegrell.

The culmination of the book is the third part, where degenerate complex Monge-Ampère equations are solved on compact Kähler manifolds by various techniques. A variational approach is presented in Chapter 11 where the complex Monge-Ampère equations under consideration appear as the Euler-Lagrange equations of certain functionals acting on finite energy classes of qpsh functions. The viscosity approach developed in the local setting in Chapter 6 is employed in the compact setting in Chapter 13. The main difficulty in solving such equations comes from the lack of smoothness of the solutions, so weak solutions have to be considered and new tools have to be developed. The partial regularity of solutions is studied in Chapters 12 and 14, where it is shown that they are Hölder continuous, and in some cases smooth, away from a divisor.

The book ends by giving in Part 4 several important applications to complex geometry of the results developed so far. Chapter 15 deals with the study of canonical metrics in Kähler geometry, the Calabi-Yau theorem, the construction of Kähler-Einstein metrics, and the Riemannian structure of the space of Kähler metrics. Chapter 16 treats the existence of singular Kähler-Einstein metrics on mildly singular varieties which are important in the Minimal Model Program.

It is worthwhile to note that this book is an extension of the lecture notes of a graduate course given by the authors at Université Paul Sabatier in Toulouse, France. Hence every chapter ends with a section of nice exercises.

This monograph covers a great deal of modern topics in several complex variables and complex geometry and gives a wide array of interesting recent applications. It will undoubtedly be a great resource for current researchers and graduate students interested in pluripotential theory and complex geometry. *Dan Coman (Syracuse)*