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Asymptotic theory of transaction costs. (English) Zbl 06694593
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This book consists of 8 chapters, 3 appendices. These lectures are devoted to portfolio optimization under proportional transaction costs $\lambda > 0$, and investigation of the asymptotic behavior, as $\lambda \to 0$. Chapter 1 deals with the models on a finite probability space. It is considered a stock price process $S = (S_t)_{t=0}^T$ in finite, discrete time, based on and adapted to a finite filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=0}^T, \mathrm{P})$. S is an R_+-valued process, and there is a bond $B = (B_t)_{t=0}^T$. By choosing B as numéraire it is assumed that $B_t \equiv 1$. Then it is introduced transaction costs $\lambda \geq 0$, i.e., the process $((1-\lambda)S_t, S_t)_{t=0}^T$ models the bid and ask prices of the stock S respectively. Fundamental theorem of asset pricing is proved, and as a corollary the superreplication theorem is derived. As an examples the one-period binomial model and one-period trinomial model are considered. In Chapter 2 it is considered the utility maximization under transaction costs in the case of finite probability space. The author fixes a utility function $U: \ D \to \mathrm{R}$. The domain D of U will be either $D = (0, \infty)$ or $D = (-\infty, \infty)$, and U is supposed to be concave, R-valued, increasing function on D. It is also assumed that U is strictly concave and differentiable on the interior of D, and that U satisfies the Inada condition $\lim_{x \downarrow x_0} U'(x) = \infty$, $\lim_{x \uparrow \infty} U'(x) = 0$, where $x_0 \in \{\infty, 0\}$ denotes the left boundary of D. The aim is to find a trading strategy maximizing expected utility of terminal wealth. The author proves the basic duality result for U and V, where $V(y) = \sup_{x \in D}[U(x) - xy]$, $y > 0$. Then the notion of shadow price is introduced and the representation of optimal trading strategy in the market S under transaction costs λ in terms of shadow price is presented. Chapter 3 is devoted to the growth-optimal portfolio in the Black-Scholes model under logarithmic utility. The frictionless case is considered. At the presence of transaction costs it is well known that one has to keep the proportion of wealth within a certain interval. It is exactly determined all the quantities of interest, e.g. the width of this corridor and the effect on the indirect utility. The dual method allows the author to calculate these quantities either in the closed form as functions of the level $\lambda > 0$ of transaction costs, or as a power series of $\lambda^{1/3}$. All the coefficients of the fractional Taylor series are explicitly computed. In the case of the Black-Scholes model, the shadow price process can explicitly be determined. Chapter 4 deals with the duality theory for financial markets under transaction costs. The author supposes that S is continuous process, and that the underlying filtration is Brownian, but do not assume that the process S is a semimartingale. The central result of this chapter establishes a polar relation between set of random variables that is attained from initial wealth $x > 0$ by trading in the stock S under transaction costs λ in an admissible way, and on the dual side, the set consists of the so-called supermartingale deflators. Chapter 5 develops a local duality theory. It is shown that several traditional assumptions in the theory of portfolio optimization can be replaced by their local versions without loss of generality with respect to the conclusions. The author gives equivalent formulation of the assumption of "no unbounded profit with bounded risk" in the frictionless setting and an analogous theorem in the setting of arbitrarily small transaction costs $\lambda > 0$. Chapter 6 is devoted to the general setting of portfolio optimization under proportional transaction costs. The central result (Theorem 6.5) of the present book shows that, under appropriate assumptions, the dual optimizer, which a priori is only a supermartingale, is in fact a local martingale. The crucial assumption underlying this result is that the process S satisfies the "two-way crossing property", a notion, which generalizes the concept of continuous martingale. In Chapter 7 the author shows that the local martingale property established in Theorem 6.5 is the key to the existence of a shadow price process. Chapter 8 is devoted to the case of exponential fractional Brownian motion. It is proved that for this model there is a shadow price process, because the fractional Brownian motion has the "two-way crossing" property. The appendices consider the polar sets, the polyhedral sets and Legendre transformation. The Bibliography consists of 77 items.

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62P05 Applications of statistics to actuarial sciences and financial mathematics
91G10 Portfolio theory

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