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★**Shape variation and optimization.**

A geometrical analysis.

English version of the French publication [MR2512810] with additions and updates.

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This book is an introduction to variational methods in shape optimization of scalar elliptic boundary value problems and spectral problems. In particular, the authors study the most important aspects of shape optimization for the Laplacian. The main topic of the book is the existence of solutions to the shape optimization problems considered. In particular, Mosco convergence in Sobolev spaces is employed in order to establish the existence of an optimal domain. Geometric properties of optimal domains are considered for specific problems. It is important to note that Mosco convergence is suitable to prove the existence of an optimal domain for the drag minimization for stationary as well as the work minimization for nonstationary compressible Navier-Stokes equations [P. I. Plotnikov and J. Sokołowski, *Compressible Navier-Stokes equations*, IMPAN Monogr. Mat. (N. S.), 73, Birkhäuser/Springer Basel AG, Basel, 2012; MR2963679].

The part of the book under review on spectral problems is interesting on its own. This is the most advanced part of the book based on recent research. In addition to the references of the book, some results on the so-called partition problem for eigenvalues can be found in [G. R. David et al., Astérisque No. 392 (2017), ii + 203 pp.; MR3706139].

Some of the results on classical shape differentiability given in Chapter 5 of the book under review could be used to establish necessary optimality conditions for shape optimization problems. A complete analysis of shape sensitivity for PDE's can be found, e.g., in [J. Sokołowski and J.-P. Zolésio, *Introduction to shape optimization*, Springer Ser. Comput. Math., 16, Springer, Berlin, 1992; MR1215733] or in [M. C. Delfour and J.-P. Zolésio, *Shapes and geometries*, second edition, Adv. Des. Control, 22, SIAM, Philadelphia, PA, 2011; MR2731611].

Recently, a general concept of topological derivatives was introduced and applied for the numerical solution of shape optimization problems in [A. A. Novotny and J. Sokołowski, *Topological derivatives in shape optimization*, Interact. Mech. Math., Springer, Heidelberg, 2013; MR3013681].

The book under review can be used as a source of information for a graduate course on variational methods in shape optimization.

{For the French original see [A. Henrot and M. Pierre, *Variation et optimisation de formes*, Math. Appl. (Berlin), 48, Springer, Berlin, 2005; MR2512810].}

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