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Compactness and stability for nonlinear elliptic equations.

Zurich Lectures in Advanced Mathematics. Zürich: European Mathematical Society (EMS) (ISBN 978-3-03719-134-7/pbk). x, 291 p. EUR 42.00 (2014).

This is an interesting book on the study of compactness and stability for nonlinear elliptic equations in the inhomogeneous context of closed Riemannian manifolds, which is a field presently undergoing great development. Some blow-up phenomena and the recent progress on the subject such that the new ideas, concepts, methods, and theories in the field are summarized in the book. As a typical equation, special attention is paid to the following stationary nonlinear Schrödinger equation on a smooth compact Riemannian $n-$manifold (M, g) without boundary:

$$-\triangle_g u + h(x)u = u^{p-1},\ p \in (2, \frac{2n}{n-2}] \text{ with } n \geq 3,$$

where $h \in \mathcal{C}$ and $\mathcal{C}$ is a given class of functions, such as L^∞, C^1 or $C^{0,\alpha}$ with $\alpha \in (0, 1)$.

There are eight chapters in the book: In Chapter 1, some model equations related to the generic stationary nonlinear Schrödinger equation which is used as a typical equation in the book are discussed; Chapter 2 is concerned with two basic variational methods for solving nonlinear elliptic PDEs, the sub and super solution method (non-variational) for the Einstein-scalar field Lichnerowicz equation; In Chapter 3, the L^p and H^1-theories for blow-up are discussed. Chapter 4 is concerned with describing several results on the opposite side of constructive analysis, where the existence of blowing-up sequences of solutions for the model equations are shown. In Chapter 5, different notions of stability for elliptic PDEs, including analytic stability and bounded stability, are discussed; Chapter 6 is concerned with bounded stability; In Chapter 7, the C^0-theory for blow-up is described; Chapter 8 is concerned with analytic stability and the notion of range of influence of blow-up points. *Huansong Zhou (Wuhan)*