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★**Emil Artin and beyond—class field theory and L-functions.**
With contributions by James Cogdell and Robert Langlands.
Heritage of European Mathematics.
European Mathematical Society (EMS), Zürich, 2015. *xiv*+231 *pp.*
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This very readable book makes valuable historical contributions to the development of class field theory, and includes good mathematical summaries of its development.

Chapter I summarizes the development of class field theory up to 1931, culminating in work of C. C. Chevalley in obtaining a completely algebraic version of class field theory, introducing the notion of idèles. Artin's work was driven by the use of zeta functions, which had appeared already in his Ph.D. thesis in 1921 [*Quadratische Körper im Gebiete der höheren Kongruenzen*, Univ. Leipzig] and in papers in 1930 and 1931 introducing Artin (non-abelian) L-functions. Chevalley's 1933 Ph.D. thesis [*Sur la théorie du corps de classes dans les corps finis et les corps locaux*, Univ. Paris] developed ideas stemming from his attendance at Artin's course on class field theory in 1931 at Hamburg. The chapter presents letters from Chevalley to Hasse on June 20, 1935, on his algebraic approach, and Hasse's reply on June 28, 1935, with English translations of each. Chevalley extended the global theory to infinite abelian extensions in 1936. His 1940 paper [Ann. of Math. (2) **41** (1940), 394–418; MR0002357] gave a complete version of class field theory dealing with finite or infinite abelian extensions, without using analytical methods, avoiding zeta functions.

Chapter II discusses Emil Artin's life in America, after his emigration from Germany in 1937, including his time at the University of Notre Dame and his permanent position at Indiana University and later at Princeton University. This chapter concerns his evolution as a great teacher, necessitated by the demands of his new positions. It includes his lecture notes on Galois Theory at the University of Notre Dame.

Chapter III discusses Artin's work with George Whaples at Indiana, including their work on simple rings [Amer. J. Math. **65** (1943), 87–107; MR0007391] introducing valuation vectors (now called adèles) and characterizing fields having a product formula for valuations [Bull. Amer. Math. Soc. **51** (1945), 469–492; MR0013145]. It reproduces Whaples' application to the Institute of Advanced Study in 1941.

Chapter IV discusses the career of Margaret Matchett, Artin's second Ph.D. student at Indiana. It presents a brief biography and includes her 1946 Ph.D. thesis [*On the zeta function for ideles*, Indiana Univ.; MR2937678] in its entirety. Her thesis was an important precursor to J. T. Tate's 1950 Ph.D. thesis [*Fourier analysis in number fields and hecke's zeta-functions*, Princeton Univ.; MR2612222; see also in *Algebraic Number Theory (Proc. Instructional Conf., Brighton, 1965)*, 305–347, Thompson, Washington, DC, 1967; MR0217026], and this book is the first time this thesis has appeared in print.

Chapter V, "L-functions and non-abelian class field theory, from Artin to Langlands", written by James Cogdell, treats the history of Artin's construction of non-abelian L-functions in 1923 through the search for a non-abelian class field theory. The article includes facsimiles of Artin's draft papers. Cogdell includes Hecke's work on L-functions in the mid-1930's attaching L-functions to modular forms. The reconciliation of these two types of L-functions was done later in the Langlands program, culminating in the Langlands functoriality principle.

Chapter VI contains two contributions of Robert Langlands. The first is a transcript of his famous letter to André Weil in January 1967 discussing Euler products and his generalization of L-functions using Euler products attached to representations of reductive groups. The second is an article ["Functorialität in der Theorie der automorphen Formen: Ihre Entdeckung und ihre Ziele", submitted] recalling his youthful work up to his recent work on functoriality.

As a complement to the book, we mention an article of Hasse on the history of class field theory, and Tate's thesis, both in [*Algebraic number theory*, Proceedings of an instructional conference organized by the London Mathematical Society (a NATO Advanced Study Institute) with the support of the Inter national Mathematical Union. Edited by J. W. S. Cassels and A. Fröhlich, Academic Press, London, 1967; MR0215665], as well as articles in the more recent volume [*Class field theory—its centenary and prospect (Tokyo, 1998)*, Adv. Stud. Pure Math., 30, Math. Soc. Japan, Tokyo, 2001; MR1846447]. *J. C. Lagarias*

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