Citations

From References: 12 From Reviews: 0

MR2790054 (2012g:22010) 22E30 22E46 32A70 46E20 47A48 47G10 Neretin, Yurii A. (A-WIENM)

★Lectures on Gaussian integral operators and classical groups.

EMS Series of Lectures in Mathematics.

European Mathematical Society (EMS), Zürich, 2011. xii+559 pp. ISBN 978-3-03719-080-7

This series of lectures takes Gaussian operators as a basis for the study of classical groups. A Gaussian integral operator has the form

$$Sf(x) = \int_{\mathbb{R}^n} \exp\left\{\frac{1}{2} \sum_{i,j} a_{ij} x_i x_j + \sum_{i,j} b_{ij} x_i y_j + \frac{1}{2} \sum_{i,j} c_{ij} y_i y_j\right\} f(y) \, dy.$$

An important feature is the exposition of the Weil representation, also associated with the names Friedrichs, Segal, Berezin, and Shale. The text presents an analytic approach to group representations rather than algebraic. The work is nicely accessible. Most calculations are at least clearly indicated if not explicitly worked out.

A list of the chapter titles gives a good idea of the approach and contents of the book:

(1) Gaussian integral operators, (2) Pseudo-Euclidean geometry and groups U(p,q), (3) Linear symplectic geometry, (4) The Segal-Bargmann transform, (5) Gaussian operators in Fock spaces, (6) Gaussian operators. Details, (7) Hilbert spaces of holomorphic functions in matrix balls, (8) The Cartier model, (9) Gaussian operators over finite fields, (10) Classical p-adic groups. Introduction, (11) Weil representation over a p-adic field, Addendum with summaries and tables.

Among the interesting features is the use of explicit matrix calculations in the spirit of L. G. Hua [Harmonic analysis of functions of several complex variables in the classical domains, translated from the Russian by Leo Ebner and Adam Korányi, Amer. Math. Soc., Providence, RI, 1963; MR0171936]. Work of J. Faraut and A. Korányi [Analysis on symmetric cones, Oxford Math. Monogr., Oxford Univ. Press, New York, 1994; MR1446489] and Farault's more recent work [J. Faraut, Analysis on Lie groups, Cambridge Stud. Adv. Math., 110, Cambridge Univ. Press, Cambridge, 2008; MR2426516; J. Faraut et al., Analysis and geometry on complex homogeneous domains, Progr. Math., 185, Birkhäuser Boston, Boston, MA, 2000; MR1727259] well complements the material of these lectures.

The text works well not only as an introduction, but as a fairly extensive survey as well. Overall it looks to be a useful reference for continued consultation and study.

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