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★**Lectures on Duflo isomorphisms in Lie algebra and complex geometry.**
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Let $\mathfrak{g}$ be a Lie algebra. Let $S(\mathfrak{g})$ be the symmetric algebra on $\mathfrak{g}$, and $U(\mathfrak{g})$ be the enveloping algebra associated to $\mathfrak{g}$. By a theorem of M. Duflo [Ann. Sci. École Norm. Sup. (4) **10** (1977), no. 2, 265–288; MR0444841], the symmetrization map $I\colon S(\mathfrak{g})\to U(\mathfrak{g})$, occurring in classical formulations of the Poincaré-Birkhoff-Witt isomorphism, can be modified to yield an isomorphism of commutative algebras on subalgebras of invariants:

$$S(\mathfrak{g})^{\mathfrak{g}} \xrightarrow{\simeq} U(\mathfrak{g})^{\mathfrak{g}}.$$

This modification is given by the pre-composition of the symmetrization map I with a certain operator $J^{1/2}$ determined by an explicitly defined power series $J(x)$ on the dual module $\mathfrak{g}^*$ of the Lie algebra $\mathfrak{g}$.

Duflo's original argument relies on Kirillov's orbit method. In [Lett. Math. Phys. **66** (2003), no. 3, 157–216; MR2062626] M. Kontsevich initiated a new approach to this subject by proving that the Duflo isomorphism theorem could be deduced from his result on the deformation quantization of Poisson manifolds. His arguments have the advantage of working in the supergraded context. In [J. Geom. Phys. **51** (2004), no. 4, 487–506; MR2085348] M. Pevzner and C. Torossian also proved, by relying on Kontsevich's ideas, that the Duflo isomorphism extends to an isomorphism of cohomology algebras

$$H^*(\mathfrak{g}, S(\mathfrak{g})) \xrightarrow{\simeq} H^*(\mathfrak{g}, U(\mathfrak{g})),$$

which include invariant algebras as degree 0 components.

The purpose of the book under review is to give a comprehensive account of this new approach of the Duflo isomorphism theorem.

The crux of the arguments, as promoted by the authors, is an interpretation of the Duflo isomorphism in terms of the Kontsevich formality quasi-isomorphism which connects the Lie algebra of polyvector fields on an Euclidean space and the Lie algebra of polydifferential operators. To be more precise, the authors deal with (formal) Q-manifolds, supergraded Euclidean spaces equipped with an odd vector field Q such that $[Q,Q]=0$, and they use a version of the Kontsevich formality theorem where the differentials include extra terms yielded by an adjoint action of such a vector field Q.

The idea is that the Chevalley-Eilenberg complex $C^*(\mathfrak{g}, S(\mathfrak{g}))$, which determines the cohomology algebra $H^*(\mathfrak{g}, S(\mathfrak{g}))$, is identified with the algebra of polyvector fields on a Q-manifold naturally associated to $\mathfrak{g}$. The Chevalley-Eilenberg complex $C^*(\mathfrak{g}, U(\mathfrak{g}))$ is, on the other hand, related to the Lie algebra of polydifferential operators. The authors precisely prove that the Duflo isomorphism has a definition in terms of a quasi-isomorphism connecting these complexes associated to Q-manifolds. This statement has a complex manifold version which the authors also explain.

The proof of the cohomological version of the Duflo isomorphism theorem given in this book mostly relies on Kontsevich's original ideas. The authors simply use a new Koszul duality argument to explain the Q-manifold interpretation of the Chevalley-Eilenberg complex $C^*(\mathfrak{g}, U(\mathfrak{g}))$.

The book is self-contained and includes introductory chapters on Lie algebra cohomology, Hochschild cohomology, and Dolbeault cohomology, as well as a comprehensive account of the definition, in terms of integrals over configurations of points in the hyperbolic plane, of the Kontsevich formality quasi-isomorphism in the context of Q-manifolds.

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