MR2918244 (Review) 46-02 41A46 46E30 65D30
Triebel, Hans (D-FSUMI)

★**Faber systems and their use in sampling, discrepancy, numerical integration.**
EMS Series of Lectures in Mathematics.
European Mathematical Society (EMS), Zürich, 2012. *viii*+107 *pp.*
ISBN 978-3-03719-107-1

This book is a continuation of a previous book from the same author [*Bases in function spaces, sampling, discrepancy, numerical integration*, EMS Tracts Math., 11, Eur. Math. Soc., Zürich, 2010; MR2667814]. It is a relatively short (around 100 pages) research monograph which in some aspects resembles a long research paper. As the author himself states, he is more interested in launching seeds for forthcoming research than exploring completely the topics chosen, which is reflected by his choice of considering only model cases or convenient restrictions for the parameters in some of the extensions, to new settings, of previously obtained results. This should be borne in mind when reading the description below.

Chapter 1 gives a good introduction to the following chapters, presenting some review material and the new spaces to be considered here, namely some weighted versions of the spaces (of Besov and Triebel-Lizorkin type and dominating mixed smoothness counterparts) dealt with in the above-mentioned previous book. In Section 1.4, after defining Faber bases and recalling representation theorems obtained previously by means of them, for example, in the estimation of the asymptotics for sampling numbers, the following motivations for the book are presented:

1. Obtain other bases, for higher smoothness spaces, in which the coefficients in the representation theorems are, similarly as when using Faber bases, obtained by evaluating the functions in finitely many points.
2. Extend the theory of Faber bases and their use in sampling, numerical integration and discrepancy to some weighted spaces.
3. Extend discrepancy estimates to smaller values of the smoothness of the spaces involved.

Afterwards, the author states explicitly what he is able to achieve regarding these three topics, distributing them between the following (and remaining) three chapters, and finishes the section by indicating what would be desirable to study next.

Chapter 2 is devoted to extending to Besov spaces on the unit interval the results on discrepancy mentioned above. The mapping property involving differentiation in Besov spaces and given in Proposition 2.1 might also have independent interest.

In Chapter 3 the author constructs Haar and Faber bases for some weighted Besov spaces on the real line and extends asymptotic results on sampling numbers to embeddings from such spaces, even for smoothness parameters higher than considered in the previous book (after extending to higher order the possibility of representing corresponding unweighted spaces by means of Faber-type systems). The asymptotics on integral numbers of the same spaces are obtained as a consequence. Next, the author considers corresponding embeddings where the weight is transferred to the target space, obtaining the same results as before. In the last part of the chapter he also obtains results on numerical integration with weights for functions in Besov spaces on the real line.

Chapter 4 is, in most respects, the counterpart of Chapter 3 for some weighted Besov spaces of dominating mixed smoothness on the plane. There are some differences though:

the range for the smoothness parameter is kept constrained as in the previous book, and some weighted Sobolev spaces of dominating mixed smoothness are also considered. Moreover, in the final section several previous results are formulated for such spaces on $\mathbb{R}^n$ (instead of the plane).

The book ends with a bibliography (including references to the pages where the works are cited), a list of symbols and an index. *António M. Caetano*