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Lecture notes on cluster algebras.

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Cluster algebras, invented by *S. Fomin* and *A. Zelevinsky* [J. Am. Math. Soc. 15, No. 2, 497–529 (2002; Zbl 1021.16017)] in order to study total positivity in algebraic groups and canonical bases in quantum groups, are a class of commutative algebras endowed with a distinguished set of generators, the cluster variables. The cluster variables are grouped into finite subsets, called clusters, and are defined recursively from initial variables through mutation on the clusters. The seed includes the cluster variables and the iterative process or mutation is codified in an exchange matrix. Cluster algebras are defined via matrices and mutations and also exchange patterns via matrices and polynomials. The book under review serves as an introductory survey on cluster algebras. It contains nine chapters and is well written.

Chapter 1 is of introductory nature and contains motivations for cluster algebras. Some combinatorial examples involving recurrences and some notations are provided. In Chapter 2, one learns about the definition of a cluster algebra. In these notes, the author focus on cluster algebras of geometric type, although more general versions have been defined. All necessary ingredients involved in the definition such as skew-symmetrizable matrices, quiver notations and exchange graphs are introduced. The first definition of a cluster algebra involves the polynomials more directly, with mutation being defined by substitutions rather than via sign-skew-symmetric matrices. In Chapter 3, the author describes this approach in detail and explains how it corresponds to the version with matrices given in Chapter 2.

Recall that a cluster algebra is said to be of finite type if it has finitely many seeds. The classification of cluster algebras of finite type is done in terms of Cartan matrices of finite type. The corresponding reflection groups and their root system play a key role in describing the cluster algebras. The author follows J.E. Humphreys' book on reflection groups and Coxeter groups to give a brief introduction to the theory of reflection groups and root system in Chapter 4. In Chapter 5, the finite type cluster algebras in terms of Dynkin diagrams are classified.

Associated to each cluster algebra of finite type is a corresponding abstract simplicial complex on the clusters known as the generalized associahedron. An overview of these results are provided in Chapter 6. Chapter 7 discusses two aspects of periodicity in cluster algebras. One is the periodicity of quivers with respect to mutations and the other is the categorical periodicity considered by *B. Keller* which led to his proof in [Ann. Math. (2) 177, No. 1, 111–170 (2013; Zbl 6146418)].

Recall that a cluster algebra is said to be of finite mutation type if the set of principal parts of mutation matrices is finite. The classification of all cluster algebras of finite mutation type associated to skew-symmetric matrices is addressed. Some of these cluster algebras are given in terms of the quivers associated to marked Riemann surfaces.

In the last chapter, the author illustrates one important example of cluster algebras. The beautiful result is that the homogeneous coordinate ring of the Grassmannian of k-subspaces of an n-dimensional space admits a cluster algebra structure. *Xueqing Chen (Whitewater)*