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★Linear forms in logarithms and applications.

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Since the remarkable results by A. Baker (starting with his [Mathematika **13** (1966), 204–216; *ibid.* **14** (1967), 102–107; *ibid.* **14** (1967), 220–228; MR0220680]), explicit lower bounds for linear forms in logarithms have transformed the study of a vast array of number theory problems. Bugeaud’s book provides a well-motivated resource for this important tool and its many applications.

Such results can be obtained for logarithms on any commutative algebraic group, but the most commonly used for applications are linear forms in complex logarithms and  $p$ -adic logarithms. This book focusses primarily on these two.

This reviewer found this book to be very accessible and, in fact, engaging. Just one of the many examples of this can be found in the discussion of the parameter  $E$  on page 11. As an introduction to the subject, as well as a refresher, it does an excellent job of presenting the topic, leaving the reader with a sound understanding of the subject and its applications. For those looking to apply linear forms in logs to their own problems, the book motivates well the nature of the proofs of the applications so that general principles or approaches are clear to the reader.

The book begins with a brief introduction, and the preface also provides informative introductory and historical material. In Chapter 2, the author states the main results on lower bounds for linear forms in logarithms, including the currently best known results due to E. M. Matveev [Izv. Ross. Akad. Nauk Ser. Mat. **62** (1998), no. 4, 81–136; MR1660150].

The application of these results begins in Chapter 3, with examples like effective irrationality measures, lower bounds for distances between  $S$ -units as well as for the greatest prime factor of terms in linear recurrence sequences, etc. For someone learning about the use of such results for the first time, this is a great entry point. Here some immediate, but very important, applications are presented, so the reader sees clearly the extraordinary power of these lower bounds. Moreover, and this continues throughout the book, the author focusses on presenting clearly the key ideas of the proofs. He also manages to do so while providing complete proofs, rather than just sketches of the proofs.

Chapter 4 covers upper bounds for the solutions of some classical families of Diophantine equations, such as  $S$ -unit equations (which are key for many of the results that follow), Thue equations, hyperelliptic and superelliptic equations and more.

Further applications are to be found in Chapters 5 through 10. I mention Chapter 9 here as it discusses simultaneous linear forms, which are often not covered, and their applications to Diophantine problems.

These applications are followed by two chapters devoted to proofs of results for linear forms in two complex or  $p$ -adic logarithms using interpolation determinants. This is a refinement of Schneider’s method. Its benefit is that, while its dependence on the author’s quantity,  $B'$ , is worse than in Matveev’s result, the constants are much better, so it is very useful for applications, especially when one wants to solve a problem completely.

The book concludes with a chapter devoted to open problems and appendices containing tools like heights and zero lemmas that are needed for the proofs of the results in the book.

Two other appealing features of this book are the inclusion of exercises and notes at

the end of most chapters.

Overall, I strongly recommend this book to people interested in this field. As I have written above, it offers a very well-presented account of this subject that genuinely offers much to readers of all levels of experience with linear forms in logarithms.

*Paul M. Voutier*