Robert Adol'fovich Minlos (1931–2018) – His Work and Legacy

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Robert Adol'fovich Minlos. (Courtesy of A. Kassian)

On 9 January 2018, the renowned mathematician Professor Robert Adol'fovich Minlos passed away at the age of 86. An eminent researcher and outstanding teacher, he was a world-renowned specialist in the area of functional analysis, probability theory and contemporary mathematical physics.

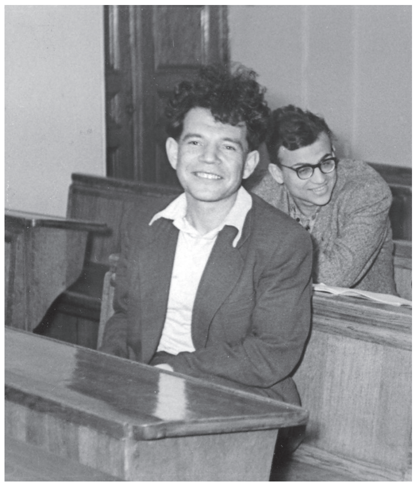
R.A. Minlos was born on 28 February 1931 into a family associated to the humanities. His father Adol'f Davidovich Miller was known as a lecturer and author of English dictionaries and manuals. His mother Nora Romanovna (Robertovna) Minlos was an historian-ethnographer. Her brother Bruno Robertovich Minlos, with a PhD in historical sciences, was a specialist in the history of Spain. This is perhaps why Robert Adol'fovich loved poetry, wrote verses himself, was a fervent theatre-goer from his school years and was seriously occupied with painting from the age of 40.

Nothing foretold a mathematical future but when he was 15, the young Robert accidentally saw a poster about the Moscow Mathematical Olympiad for schoolchildren. He participated in it, obtained the second prize and, inspired by that, began to attend the school club led by E.B. Dynkin. In 1949, Robert joined the Faculty of Mechanics and Mathematics of the Moscow State University. He continued to participate in Dynkin's seminar, which, together with A.S. Kronrod's seminar, had a great influence on him as an undergraduate student.

R.A. Minlos prepared his first scientific paper (equivalent to a Master's degree thesis) in 1950 while participating in the Moscow State University seminar on the theory of functions of a real variable (under the leadership of A.S. Kronrod). However, the real scientific interests of the young mathematics student began to form after he became acquainted to I.M. Gelfand. Their joint publication "Solution of the equations of quantum fields" (*Doklady Akad. Nauk SSSR*, n.s., 97, 209–212, 1954) became Minlos' diploma thesis in mathematics. It was devoted to the *functional* or, in mathematical physics language, the *path* integral, which has a direct relation to quantum physics.

As Minlos himself admitted: "My further life in mathematics was predetermined by that work because I was subsequently mainly occupied with mathematical physics." There were, nevertheless, more works on random processes, on measure theory and on functional analysis. Very soon, one of his papers, "Extension of a generalised random process to a completely additive measure" (*Doklady Akad. Nauk SSSR*, 119, 439–442, 1958), brought Minlos to worldwide fame. It became the basis of his Candidate (equivalent to PhD) Dissertation "Generalised random processes and their extension to a measure", which was published in *Trudy MMO*, 8, 497–518, 1959. This result, which is important for the theory of random processes, as well as for functional analysis, is now known as the *Minlos theorem* on the extension of cylindrical measures to Radon measures on the continuous dual of a nuclear space, i.e. the continuation of a process to a measure on spaces adjoint to nuclear spaces.

The connection of Minlos to mathematical physics at that time manifested in the publication (jointly with I.M. Gelfand and Z.Ya. Shapiro) of the monograph *Representations of the rotation and Lorentz groups and their applications* (1958), which was later translated from the Russian by Pergamon, London, in 1964. Note that the monograph appeared in 1958, just at the time when the need for physicists to understand representation theory was strongly motivated by the discovery of elementary particle symmetries, as well as the role of their spins and symmetries related to relativistic Lorentz transformations.



Seminar of F. A. Berezin and R. A. Minlos at the Faculty of Mechanics and Mathematics – MSU, 1959. (Courtesy of N.D. Vvedenskaya)

From 1956 to 1992, R.A. Minlos was employed by the Department of the Theory of Functions and Functional Analysis of the Faculty of Mechanics and Mathematics at Moscow State University (MSU). In that period, there was a need to organise a joint seminar with F.A. Berezin, primarily to discuss the mathematical problems of quantum mechanics and quantum field theory.

A real advance of activity in the field of mathematical physics at the Faculty of Mechanics and Mathematics of MSU was achieved with the organisation in 1962 by R.A. Minlos and R.L. Dobrushin of a seminar on statistical physics. It soon became widely known in the Soviet Union and abroad as the *Dobrushin–Malyshev–Minlos–Sinai* seminar. The quantum aspects of statistical mechanics at the seminar were primarily associated to the name of R.A. Minlos. The seminar lasted until 1994 and had a huge impact on the world of modern mathematical physics. Almost all the celebrated specialists in the field visited Moscow during the lifespan of the seminar. Besides traditional scientific contacts with Socialist European countries, a fruitful collaboration was also established with colleagues from other countries. However, the most intensive contacts were within the country, involving almost all the republics. For example, many of Minlos' PhD students came from Uzbekistan.

The beginning of the 1960s was extremely fruitful for Robert Adol'fovich. Initially, there were new results obtained jointly with L.D. Faddeev on the quantum mechanical description of three particles (1961). This was followed by two articles devoted to the study of the thermodynamic limit in classical statistical physics (1967). There, R.A. Minlos suggested the first rigorous mathematical definition of the limiting Gibbs distributions for an infinite system of interacting classical particles and also analysed the properties of such distributions (*Funct. Anal. Appl.*, 1, 140–150 and 206–217, 1967). His approach was very close to the classical Kolmogorov construction of random processes (fields). This result anticipated the origin of the Markovian understanding of Gibbs random fields in the sense of Dobrushin–Lanford–Ruelle (1968).

The result (together with Ya.G. Sinai) of the appearance of phase separation in lattice systems at low temperatures (*Math. USSR-Sb.*, 2, 335–395, 1967; *Trudy MMO*, 17, 213–242, 1967, and 19, 113–178, 1968) was of fundamental importance for the mathematical theory of phase transitions. It formed the basis of Minlos' doctoral dissertation, which he submitted for habilitation in 1968. In another joint work with Ya.G. Sinai (*Theor. Math. Phys.*, 2, 167–176, 1970), the foundation was laid for a new approach to the study of spectral properties of many-particle systems. In combination with cluster expansions, this approach drove significant progress in the description of properties of such infinite systems, including the spectrum of elementary particles of quantum fields and the mathematical description of the *quasi-particle* picture in statistical physics.

The new powerful method of cluster expansions was, from the very beginning, a central one in the list of interests of Robert Adol'fovich. The results of a large series of papers in this topic by R.A. Minlos, V.A. Malyshev and their students have been summarised in two monographs: *Gibbs Random Fields: Cluster Expansions* (Springer 1991, translation of the 1985 Russian edition) and *Linear Infinite-particle Operators* (Amer. Math. Soc. 1995, translation of the 1994 Russian edition). As was outlined in the book *Gibbs Random Fields*, the method of cluster expansions provides, besides a construction of the limiting Gibbs measure, a cluster representation of the projections of the limiting Gibbs measure onto bounded regions.

A famous peculiarity of the Dobrushin–Malyshev–Minlos–Sinai seminar was not only its duration of about four hours, which was amazing for foreign guests, or the assertive directness in communicating with lecturers but also the opportunity to obtain from the discussions some interesting problems to be solved. In essence, the seminar was functioning as a *machine*, generating questions and a possible way to convert them into answers. Robert Adol'fovich was always one of the sources of interesting questions and open problems. The list of projects thus originated includes, for example, cluster expansions and their applications to the problem of uniqueness/non-uniqueness of Gibbs states, the quantum three-particle problem, the Trotter product formula for Gibbs semigroups, the study of infinite-particle operators spectra, the analysis of the quasi-particle picture in statistical physics and many others.

The Dobrushin–Malyshev–Minlos–Sinai seminar had a tradition of studying and discussing new important publications on mathematical results and problems in statistical physics. The Trotter product formula problem for Gibbs semigroups was originated after discusssing the Pavel Bleher report about new techniques, *reflection positivity* and *infrared bound estimates* (launched by Fröhlich–Simon–Spencer (1976–1978) to prove the existence of phase transitions). In the case of quantum systems, this technique involves the Trotter product formula approximation of the Gibbs density matrix. In this context, Robert Adol'fovich posed a question about the topology of convergence of the Trotter product formula because, to obtain the infrared bound, one has to interchange the trace and the limit of the Trotter approximants. In fact, this operation is not harmful for quantum spin systems since the underlying Hilbert spaces are finite-dimensional but it does produce a problem, for example, in the case of *unbounded* spins. A typical example is the problem of the proof of infrared bounds for the case of structural phase transitions in one-site double-well anharmonic quantum crystals with harmonic interaction between sites (Fröhlich, 1976). Then, the interchange is possible only when the Trotter product formula converges in trace-norm topology. For a particular case of anharmonic quantum crystals, the convergence of the Trotter product formula in the trace-norm topology was proved via the Feynman-Kac representation for Schrödinger (Gibbs) semigroups. The abstract result, which also includes a generalisation to trace-norm Trotter–Kato product formula convergence, is due to H. Neidhardt and V.A. Zagrebnov (1990). So, the answer to the question posed by Robert Adol'fovich was solved affirmatively in favour of trace-norm topology for the case of Gibbs semigroups.

At the end of the 1990s, Robert Adol'fovich returned to the question of quantum phase transitions, in the area of anharmonic quantum crystals (which was already well-known to him) but from the opposite direction. It is known that in contrast to classical systems, phase transitions in their quantum analogues may disappear due to intrinsic quantum fluctuations, which may lead to important tunnelling in the double-well potential. A particular manifestation of that is the elimination by these fluctuations of the order parameter even at zero temperature, whereas it is non-zero in the classical limit when the Planck constant $\hbar = 0$. A typical example is the above structural phase transition in one-site double-well potential anharmonic quantum crystals with harmonic interaction between sites for particles of mass m in each site. Moreover, experimental data for crystals close to this model manifest a so-called "isotopic effect": the order parameter of the structural phase transition for samples with light masses $m < m_c$ disappears, a fact which warms up interest in the mathematical aspect of this phenomenon.

R.A. Minlos with co-authors N. Angelescu and V.A. Zagrebnov on a visit to Dublin Institute for Advanced Studies, 2000.

During his visits to Dublin, Leuven and Marseilles, Robert Adol'fovich, in collaboration with E.A. Pechersky, A. Verbeure and V.A. Zagrebnov, addressed the proof of the existence of a critical mass m_c such that below this threshold the quantum state of the system is in a certain sense *trivial*, or at least the order parameter is trivial. In two papers, with A. Verbeure and V.A. Zagrebnov (2000) and then with E.A. Pechersky and V.A. Zagrebnov (2002), R.A. Minlos proposed using cluster expansion techniques for the small parameter $\xi = \sqrt{m}/\hbar$. Then, the classical limit corresponds to $\xi \to \infty$ and the quantum regime, with zero order parameter for any temperature, corresponds to $\xi < \sqrt{m_c}/\hbar$. Since the structural phase transition in the model manifests as the displacement order parameter, these papers consider projection of the full quantum state on the commutative coordinate *-subalgebra $\mathfrak{A}_q$ of bounded functions of displacements on the lattice. Then, the Feynman–Kac–Nelson formula for the Gibbs semigroup kernel allows one to show that this projection reduces to a classical ensemble of weakly interacting (for $m < m_c$) Ornstein–Uhlenbeck trajectories. Using the cluster expansion technique, the exponential mixing of the limit state with respect to the lattice group translations was proven for all temperatures, including zero, if $m < m_c$ (2000). To check that in this domain of light masses (high *quantumness*) the order parameter is zero for all temperatures, including the ground state, R.A. Minlos and his coauthors, in the 2002 paper, used the external sources h conjugated to the local displacements instead of localising the trajectories boundary conditions, as in the 2000 paper. This allows the analysis, for any temperature θ, of the free-energy density function $f(\theta,h)$ for free or periodic boundary conditions. It is proved in the 2002 paper that there exists a radius $h_0(m)$ such that $h \mapsto f(\theta,h)$ is holomorphic in the disc $\{h \in \mathbb{C}: |h| < h_0(m)\}$ for any $\theta \geq 0$ if $m < m_c$. Moreover, the Gibbs expectations: $h \mapsto \langle A \rangle(\theta,h)$, are holomorphic in the same disc for any bounded operator A of a quasi-local *-algebra $\mathfrak{A}$ of observables. The analyticity, in particular, yields that the displacement order parameter is equal to zero for $h = 0$ and for any temperature $\theta \geq 0$ if $m < m_c$.

This was also a period when, during his visits to KU Leuven and CPT Marseilles, Robert Adol'fovich got into an argument with A. Verbeure about the mathematical sense of the notion of *quasi-particles* in many-body problems and of the *corpuscular* structure of infinite system excitations. In the framework of quantum statistical mechanics, an attractive way to promote this notion was based on the non-commutative central limit theorem for *collective* excitations. This concept yields a plausible (for physics) picture of *boson* quasi-particles excitations (phonons, magnons, plasmons, etc.) in the corresponding Fock spaces (A. Verbeure et al. (1995)).

On the other hand, in their book *Linear infinite-particle operators*, V.A. Malyshev and R.A. Minlos proposed the description of a quasi-particle picture based on the construction by cluster expansions of the lower branches of the spectrum of infinite many-body systems with good clustering. This idea goes back to the paper by R.A. Minlos and Ya.G. Sinai "Investigation of the spectra of some stochastic operators arising in the lattice gas models" (1970). There, a new approach to studying the spectral properties of the transfer matrix in general lattice models at high temperatures was developed. For translation-invariant systems, the lowest branch of the spectrum enumerated by momentum corresponds to one-quasi-particle excitations above the ground state. Then, in the simplest case, the energy of these excited states is completely defined by the momentum. This is called a dispersion law for quasi-particles, which is also well-known for boson quasi-particles. If the system possesses a good clustering, one can construct separated translation-invariant two-, three- and more (interacting) quasi-particles excited states, which are combinations of branches with bands of continuum spectra. Robert Adol'fovich called this property of excitations "The 'corpuscular' structure of the spectra of operators describing large systems" (the title of his paper in *Mathematical Physics* 2002, Imperial Coll. Press, 2000).

The technique developed by V.A. Malyshev and R.A. Minlos allowed the study of the *corpuscular* structure of generators of stochastic dynamics. Their approach was also applied to generators of stochastic systems: Glauber dynamics, the stochastic models of planar rotators, the stochastic Ising model with random interaction and other lattice stochastic models with compact and non-compact spin spaces, as well as stochastic dynamics of a continuous gas and other stochastic particle systems in the continuum. Using this technique, one can find spectral gaps and construct lowest one-particle invariant subspaces of the generator that determine the rate of convergence to the equilibrium Gibbs state. Moreover, it also allowed the study, in detail, of the spectrum branches of infinite-particle operators on the leading invariant subspaces and, in particular, the construction of two-particle bound states of the cluster operators. These results led to the understanding that a wide class of linear infinite-particle operators of systems in a regular regime have a corpuscular structure.

Visiting Leuven and Marseilles, Robert Adol'fovich proposed elucidating the concept of corpuscular structure of spectral branches for several particular models on the lowest level of one-particle elementary excitations. This programme was performed in papers with N. Angelescu and V.A. Zagrebnov (2000, 2005) and then all together with J. Ruiz (2008), for lattice models and for polaron-type problems. More activity and results in this direction were due to intensive collaboration of Robert Adol'fovich with the group of H. Spohn, where he studied spectral properties of Hamiltonians for quantum physical systems, in particular for Nelson's model of a quantum particle coupled to a massless scalar field.

Another long-term and fruitful collaboration of R.A. Minlos was with the Bielefeld group, essentially with Yu.G. Kondratiev and his co-authors and pupils. Firstly, they generalised the original method (adapted for lattices) to functional spaces to control general particle configurations. This allowed the extension of their analysis from lattice to *continuous* systems. In the paper "One-particle subspace of the Glauber dynamics generator for continuous particle systems" (2004), they studied, in detail, the spectrum of the generator of Glauber dynamics for continuous gas with repulsive pair potential. To this end, the invariant subspaces corresponding to the corpuscular structure were constructed.

The ideas and technical tools elaborated in this paper were used in a number of other projects (Yu. Kondratiev, E. Zhizhina, S. Pirogov and O. Kutoviy) on equilibrium and non-equilibrium continuous stochastic particle systems. This, in particular, concerns a delicate continuous models problem of thermodynamic limit. One unexpected application of this technique concerns image restoration processing. Robert Adolfovich, in collaboration with X. Descombes and E. Zhizhina, actively participated in the INRIA project on the mathematical justification of a new stochastic algorithm for object detection problems. The result was summarised in the article "Object extraction using stochastic birth-and-death dynamics in continuum" (2009).

In addition to the Dobrushin–Malyshev–Minlos–Sinai seminar in the 1970s, there was also a regular tutorial seminar, which was led by Robert Adol'fovich once a week. This was a very good opportunity to learn elements of topological vector spaces, in particular the Minlos theorem about the extension of a generalised random process to a measure on spaces adjoint to nuclear spaces. The seminar also covered elements of mathematical statistical physics in the spirit of the famous "Lectures on statistical physics" in *Uspekhi Math. Nauk* (1968). These lectures of Robert Adol'fovich very quickly became a textbook for many students and scientists interested in mathematical statistical physics. In these lectures, Robert Adol'fovich systematically used the notion of configuration space, which appeared in his earlier work, where he gave the mathematical definition of the limiting Gibbs measure as a measure on the space of locally finite configurations in $\mathbb{R}^d$. This concept is technically very useful and is close to modern random point process theory.

R.A. Minlos, Ya.G. Sinai and R.L. Dobrushin were often invited by the Yerevan State University and the Institute of Mathematics of the Armenian Academy of Sciences to give lecture courses on statistical mechanics. They all had PhD students working at the Institute of Mathematics in Yerevan. This was the main motivation for the Institute of Mathematics to organise regular conferences (every 2–3 years) in Armenia under the name "Probabilistic methods in modern statistical physics". The first one was held in 1982 and the last one in 1988, three years before the collapse of the Soviet Union.

R.A. Minlos with participants of the conference "Probabilistic methods in modern statistical physics", Yerevan, Lake Sevan, 2006.

The conferences restarted in 1995 at the international level. Robert Adol'fovich participated (as a rule, with his students) in all of them, including the conference in Lake Sevan in 2006. He always supported the conferences in Armenia by being a permanent member of the programme committee and one of the main speakers, formulating new problems and generating interesting ideas, questions and discussions. Unfortunately, he was not able to participate at the conferences after 2006.

In the early 1990s, Robert Adol'fovich began his collaboration with Italian institutions and mathematicians. He was a guest of the Department of Mathematics at the University of Rome "La Sapienza" many times

R.A. Minlos with participants of the conference: S. Lakaev, V. Zagrebnov, H. Suqiasian and B. Nahapetian, Yerevan, Lake Sevan, 2006.

and he also visited other institutions in Trieste, Naples, L'Aquila and Camerino. During his stay at "La Sapienza", he read a course on the mathematical foundations of statistical mechanics, which was published as a book by the American Mathematical Society in 2000 under the title *Introduction to mathematical statistical physics*'. In Rome, he began a collaboration with C. Boldrighini and A. Pellegrinotti on models of random walks (RW) in interaction with a random environment fluctuating in time ("dynamic environment").

At that time, several important results on random walks in a fixed environment had already been obtained, by Solomon, Kesten, Sinai and others, but very little was known for dynamic environments. Following the usual terminology, the behaviour of RW for a fixed choice of the history of the environment is called "quenched" and its distribution induced by the probability measure of the environment is called "annealed". A first result had been obtained by C. Boldrighini, I.A. Ignatyuk, V.A. Malyshev and A. Pellegrinotti on the annealed model of a discrete-time random walk on a d-dimensional lattice in mutual interaction with a dynamic random environment. Robert Adol'fovich proposed applying the results that he had obtained, together with V.A. Malyshev and their students, on the spectral analysis of the transfer matrix for perturbed homogeneous random fields. The approach proved to be very fruitful and in the following years 1993–1996, several results (C. Boldrighini et al. (1994)) were obtained on the annealed RW, on the convergence to a limiting measure for the field "as seen from the particle", on the decay of the space-time correlation for the random field in interaction with the RW and on the RW of two particles in mutual interaction with a random environment.

It was then possible, with the help of some tools of complex analysis of which Robert Adol'fovich had a deep knowledge, to deal with the quenched model of the RW. After the first results of a perturbative approach (C. Boldrighini et al. (1997)), a complete non-perturbative answer could be obtained for the case when the components of the dynamic environment $\xi = \{\xi(x,t):(x,t) \in \mathbb{Z}^d \in\times \mathbb{Z}\}$ are independent, identically distributed random variables (C. Boldrighini et al. (2004)). Unlike the case with fixed environment, the quenched RW in dynamic environment behaves almost surely as the annealed RW in all dimensions $d \geq 1$. In low dimension $d=1,2$, the random correction to the leading term of the RW asymptotics is of an "anomalous" large size. A quenched local limit theorem was also obtained, with an explicit dependence on the field as seen from the particle. The results were then extended to models of directed polymers in dimension $d>2$ below the stochastic threshold. Results for a quenched model of RW in a dynamic environment with Markov evolution were also obtained in dimension $d>2$ by cluster expansion methods (C. Boldrighini et al. (2000)). Further results on models of RW in a dynamic environment were obtained by Zeitouni, Rassoul-Agha, Liverani, Dolgopyat and others.

Later on (in collaboration with F.R. Nardi), it was possible to derive Ornstein–Zernike asymptotics for the correlations of a Markov field in interaction with a RW (C. Boldrighini et al. (2008)) and also for a general "two-particle" lattice operator (C. Boldrighini et al. (2011)).

R.A. Minlos with A. Pellegrinotti and C. Boldrighini, Yerevan, 2006.

In the last few years, the interest of Robert Adol'fovich in the study of random walks in a dynamic random environment did not fade and several difficult problems were solved. They concern extensions to continuous space (C. Boldrighini et al. (2009)), to continuous time and to the case of long-range space correlations for the environment (in collaboration with E.A. Zhizhina).

Robert Adol'fovich was a wonderful teacher and a patient and wise mentor. Directness, accessibility and enthusiasm attracted numerous students and followers to him. Many of his later PhD students made their first acquaintance with special branches of mathematics and mathematical physics due to the tutorial seminar at the Faculty of Mechanics and Mathematics at MSU. There, they benefited from direct generous contact with the *Master*. This student seminar was combined with lectures and scientific seminars guided by Robert Adol'fovich, together with F.A. Berezin and then with V.A. Malyshev. The lecture notes gave rise to many nice and popular tutorial books, for example *Introduction to mathematical statistical physics*, published by R.A. Minlos in Univ. Lect. Series, Vol. 19, AMS 2000. Many of Minlos' former students successfully continue research in different branches of mathematics and mathematical physics, for example: S.K. Poghosyan and E.A. Zhizhina – spectral theory of infinite systems and mathematical problems of statistical mechanics; S. Lakaev – operator spectral theory and mathematical quantum mechanics; A. Mogilner

– mathematical biology; E. Lakshtanov – infinite particle systems; and D.A. Yarotsky – random processes and spectral theory of infinite systems.

Besides the students and the tutorial seminar, Robert Adol'fovich was in contact with followers and co-authors assisting the crowded Dobrushin–Malyshev–Minlos–Sinai research seminar. There, Minlos initiated a number of projects, often related to discussions during the seminar. Always attentive and gentle, Robert Adol'fovich shared his enthusiasm to encourage followers in solving the problems.

In this way, Minlos launched the project "Cluster expansions" with V.A. Malyshev. In fact, this happened by accident when they were both in a lift while attending a seminar in the tall main MSU building. Less unusual were the origins of the projects "On the spectral analysis of stochastic dynamics" with E.A. Zhizhina and "Gibbs semigroups" with V.A. Zagrebnov, which in fact started from questions formulated during and after the seminar. The origins of many of them were due to active contacts made by Robert Adol'fovich travelling to other research centres. This is, for example, the case for the project "Application of the spectral analysis of the stochastic operator to random walks in dynamic random environments" with C. Boldrighini and A. Pellegrinotti and "Spectral properties of multi-particle models" with H. Spohn, as well as "Infinite dimensional analysis" and "Stochastic evolutions in continuum" with Yu.G. Kondratiev.

To his students and collaborators, Robert Adol'fovich was a *Master*, who, like a brilliant sculptor, could create a mathematical masterpiece from a shapeless block by cutting off the excess. Sometimes, it brought not just a feeling of amazement but a sense of miracle when, as a result of some incredible expansions, evaluations, virtuoso combinations with various spaces and other technical refinements, complex infinite-dimensional and infinite-particle systems took an elegant, precise and easily understandable form.

In this connection, problems related to the theory of operators and to quantum physics should be especially noted. This theme began in his joint paper with I.M. Gelfand and, since then, it has permanently been the focus of Minlos' attention. In 2010, together with his old co-author and friend V.A. Malyshev, he turned to a fundamental question in quantum chemistry: what is the interaction between atoms? (*Theoretical and Mathematical Physics*, 162, 317–331, 2010.) However, his favourite subject since the 1960s and until recently has been the quantum three-body problem and point interaction. A long paper ("A system of three quantum particles with point-like interactions", Russian Math. Surveys, 69, 539–564, 2014) was published by R.A. Minlos on this topic.

A recent paper by Robert Adol'fovich was dedicated to another of his favourite subjects: the random walk in a random environment ("Random walk in dynamic random environment with long-range space correlations", *Mosc. Math. J.*, 16:4 (2016), 621–640, with C. Boldrighini and A. Pellegrinotti). His very last manuscript (with C. Boldrighini, A. Pellegrinotti and E.A. Zhizhina) was also on this subject: "Regular and singular continuous time random walk in dynamic random environment".

Robert Adol'fovich selflessly served science and, in everyday life, was a generous and friendly person. He gladly shared his enthusiasm and energy with his students and colleagues. In addition to the accuracy of reasoning and complicated techniques involved, there is always a beautiful idea and harmony in his works. It is interesting to mention his response to the question of Natasha Kondratyeva: "What three mathematical formulas are the most beautiful to you?" He gave the answer: "The Gibbs formula, the Feynman–Kac formula and the Stirling formula." And those are the formulae that were widely used by Robert Adol'fovich in his works.

R.A. Minlos, Moscow 2016 (Courtesy of E. Gourko).

Robert Adol'fovich was notable for his figurative Russian language and good wit, often with subtle mathematical humour. In the 1980s, in a conversation with Roland Dobrushin at the Fourth Vilnius Conferences on Probability Theory and Mathematical Statistics (1985), he expressed his doubt that "the life of a Soviet citizen is *complete* with respect to the *norm* of the anti-alcohol campaign". A campaign was ongoing at that time in the country under the slogan "Sobriety is the norm of our life!" and was visible everywhere on white-red streamers. Since then, this allusion to the completeness of life and normed spaces has entered into the folklore of the mathematical community.

Always surrounded by relatives and intimates, and also by loving pupils, colleagues and friends, Robert Adol'fovich Minlos lived a complete life. In each of those who knew Robert Adol'fovich, he left a bright drop of memory of himself.