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**Errata and Addendum to "Existence Result for the One-Dimensional Full Model of Phase Transitions", Zeitschrift für Analysis und ihre Anwendungen 21 (2002) 335–350**

**Errata**

- We shall ask $\partial_x \chi_0(0,\cdot) = \partial_x \chi_0(\ell,\cdot) = 0$ a.e. in $(0,T)$ in $(3.1)_4$ and in Remark 3.2.

- In $(3.2)_2$ and $(3.2)_3$ we actually prove the stronger regularities $L^\infty(0,T;W)$ and $L^\infty(0,T;H)$, respectively.

- In $(4.4)_2$ one also obtains $\chi_\epsilon \in L^\infty(0,T;H^3(\Omega))$.

- The first term in the left-hand side of (5.1) is $\|\partial_t \tilde{\chi}\|^2_{L^2(0,t;H)}$.

- The final right-hand side of (5.2) is

$$\int_0^t \|\partial_t \chi_1\|_{L^\infty(\Omega)} \|\tilde{\theta}\|^2 + \int_0^t \|\theta_2\|^2_{L^\infty(\Omega)} \|\tilde{\theta}\|^2 + \frac{1}{4}\int_0^t \|\partial_t \tilde{\chi}\|^2$$
$$+ \int_0^t \|\partial_t \chi_1 + \partial_t \chi_2\|^2_{L^\infty(\Omega)} \|\tilde{\theta}\|^2 + \frac{1}{4}\int_0^t \|\partial_t \tilde{\chi}\|^2.$$

- Page $341^2$: "belong to $L^1(0,T)$".

- The proof of Proposition 6.1 shall be modified as follows. For all $\tau \in (0,T]$ and a fixed constant $\kappa \geq 2\|\partial_x \theta_0\|$ we define

$$Y(\tau) = \{ f \in H^1(0,\tau;H) \cap L^\infty(0,\tau;V) \mid f(0) = \theta_0 \text{ and }$$
$$\|\partial_t f\|^2_{L^2(0,\tau;H)} + \|\partial_x f\|^2_{L^\infty(0,\tau;H)} \leq \kappa^2 \}.$$

  Hence $S$ maps $Y(\tau)$ into $H^1(0,\tau;H) \cap L^\infty(0,\tau;V)$ and it is compact and continuous. It is straightforward to exploit the dependence of $C$ on $\kappa$ in (6.5) - (6.7), suitably rewritten on the time interval $(0,\tau)$. In particular, one obtains that, for sufficiently small $\tau$, the operator $S$ takes values in $Y(\tau)$, namely Problem $1\epsilon$ has a *local in time* solution $(\theta_\epsilon, \chi_\epsilon)$. Following the arguments of Section 8 up to estimate (8.8) and then recovering the bounds of Section 6, we obtain that $\partial_x \chi_\epsilon(0,\cdot) = \partial_x \chi_\epsilon(\ell,\cdot) = 0$ a.e. in $(0,\tau)$

and $(\theta_\epsilon, \chi_\epsilon)$ is bounded in $H^1(0,T^*;H\times H)\cap L^\infty(0,T^*;V\times H^3(\Omega))$, where $T^*$ is the supremum of all $\tau$ for which there exists a solution to Problem $1\epsilon$ in $[0,\tau)$. Hence, standard prolongation arguments ensure that $(\theta_\epsilon, \chi_\epsilon)$ may be extended to a global solution to Problem $1\epsilon$ on the whole interval $(0,T)$.

- Page $342^{14}$ is $C\big(1 + \|\chi_0\|^2 + \|f_n\|^2_{L^2(0,t;H)} + \int_0^t \|\partial_t u_n\|^2_{L^2(0,s;H)}\, ds\big)$.

- The last term in the right-hand side of page $342_1$ is $\int_0^t \|(\partial_t u_n)^2 - v_n \partial_t u_n\| \, \|\partial_t v_n\|$.

- The subscript $\epsilon$ is missing in the second estimate of Subsection 8.2. Moreover, we shall modify it as $C\int_0^T \big(\|\partial_x(\theta_\epsilon^{1/2})\|^2_{L^1(\Omega)} + \|\theta_\epsilon\|_{L^1(\Omega)}\big) \le C\big(1 + \int_0^T \big(\int_\Omega \big|\frac{\partial_x\theta_\epsilon}{\theta_\epsilon^{1/2}}\big|\big)^2\big)$.

- In Subsection 8.5 we shall just integrate on $\Omega$ and exploit $(8.6)_2$ in order to get that $\chi_\epsilon$ is bounded in $L^\infty(0,T;W)$. Whence, relation (8.10) should be suitably modified as $\|\beta_\epsilon(\chi_\epsilon)\|_{L^\infty(0,T;H)} \le C$.

- The convergences in $(9.1)_4 - (9.1)_5$ are weak star in $W^{1,\infty}(0,T;H) \cap L^\infty(0,T;W)$ and $L^\infty(0,T;H)$, respectively.

- Page 349, line 5: “liminf”.

- References [5, 7, 9, 11] shall be updated as follows:


  - [5] Bonfanti, G, Frémond, M. and F. Luterotti: *Local solutions to the full model of phase transitions with dissipation.* Adv. Math. Sci. Appl. 11 (2001), 791 – 810.

  - [7] Colli, P., Luterotti, F., Schimperna, G. and U. Stefanelli: *Global existence for a class of generalized systems for irreversible phase changes.* Nonlin. Diff. Equ. Appl. (NoDEA) 9 (2002)2, 255 – 276.

  - [9] Laurençot, Ph., Schimperna, G. and U. Stefanelli: *Global existence of a strong solution to the one-dimensional full model for irreversible phase transitions.* J. Math. Anal. Appl. 271 (2002), 426 - 442.

  - [11] Luterotti, F., Schimperna, G. and U. Stefanelli: *Global solution to a phase field model with irreversible and constrained phase evolution.* Quart. Appl. Math. 60 (2002)2, 301 – 316.

**Addendum.**

With respect to $(3.2)_7$ we are actually in the position of proving the stronger result

**Lemma 1.** *Let $(\theta, \chi, \eta)$ be a solution to Problem 1. Then there exists a constant $\theta_* > 0$ such that $\theta \geq \theta_*$ a.e. in $Q$.*

**Proof.** It suffices to repeat the argument of Lemma 7.1 with the choice $\Theta(t) := \big(\theta(t) - \theta^* \exp\big(-\|\partial_t \chi\|_{L^1(0,t;L^\infty(\Omega))}\big)\big)^- \quad (t \in (0,T))$. Indeed, we are still in the position of applying both the same sign considerations and the Gronwall lemma and deduce that $\theta \geq \theta^* \exp\big(-\|\partial_t \chi\|_{L^1(0,T;L^\infty(\Omega))}\big) =: \theta_* > 0$ a.e. in $Q$.