On How to Climb the Highest Mountains – Experiences During a Week in the Heidelberg Laureate Forum

Demian Nahuel Goos (Universidad Nacional de Rosario, Rosario, Argentina)

The author shares his thoughts and experiences of the 5th Heidelberg Laureate Forum in 2017, during which he conversed with numerous mathematics and computer science laureates.

Introduction

A gale is blowing. Whiteness everywhere. A heavy blizzard has reigned over the landscape for two days, with no end in sight. The snow does not fall but rather hits you in the face, and the wind is as loud as the engines of a Boeing 747-400. It's cold. It's freezing. However, you no longer feel it. You lose your senses: up is down and down is up. What can you see? Nothing. Only wilderness. Both the peak that was recently left behind and home feel as far away as far can get. Surely, even though people think that getting to the peak is the goal, they easily overlook the fact that this is the most dangerous moment. Exhausted, stunned, overwhelmed and alone. It's you and the mountain – and nothing more. You have to get back. Only then have you completed the adventure. Getting back is the goal – nothing less, nothing more.

Using the words of Hans Kammerlander, an Italian mountaineer who specialised in fast climbing (setting a series of world records in this area):

> *We own a mountain only when back in the vale.*
> *Until then, we are the ones who belong to the mountain.*

As a beautiful and poetic way to describe this feeling, this phrase acquires a deeper significance when one learns that Kammerlander himself had lost friends during an expedition.

However, what does this have to do with maths? How could the life of a mathematician be remotely comparable to the struggle for life in extreme circumstances? This is certainly possible. I like to compare the mountains to mathematical problems, conjectures or – if a proof is completed – theorems:

> *We own a theorem only when a proof is completed.*
> *Until then, we are the ones who belong to the problem.*

This describes in many ways what mathematicians deal with. The solution to a problem can be compared to the peak of the mountain: it is of no value to us if we do not prove that this conjecture is true. Without the proof, it is comparable to a mountaineer who decides to stay back and remain on the mountain. An unfinished quest. Finding the solution is not the goal but a first sense of achievement, and it is followed by the fundamental task of proving its trueness.

I frequently hear that people compare the work of a mathematician to playing a game. After all, you set up some rules – our precious axioms – and start working with them. This visualises very well the fact that most, if not all, mathematicians love what they do and truly do enjoy it. But a game is no serious business – and maths truly is! Mathematicians do not play with toys but they face important and challenging questions. Mathematicians are adventurers, the mountaineers of truth. Similar to a mountaineer, to face the difficult problems, you first need to gain some experience with many small mountains or you will not succeed. The big mountains are left for only the greatest adventurers among us. With each mountain, you become more experienced. Sometimes you fail and do not succeed, and other times, it takes different attempts until the quest is completed. Just like mountain climbing, it is best done in small groups but not alone.

However, this metaphor has more facets. It also describes what goes through a mathematician's mind. A mountaineer enjoys climbing every mountain he comes across. Big mountains, small mountains – he enjoys every challenge, regardless of whether he already knows that he will succeed. Honestly, what mathematician does not enjoy the simple joy of solving puzzles in the newspaper or of facing some problems that undergrads deal with? Further, once we start a problem, there is no escape. Our mind will not let off until it is solved. We cannot help but think of our work while showering, while walking down the street or before we go to bed. They catch us and won't let us go. Until it is solved, we are the ones who belong to the problem.

This is the tale of those who got to the peak, saw the world from above and came back to tell the story, those

adventurers who faced the highest mountains, the trickiest quests, those who know the *Aconcagua* from within, survived the *Himalayas* or even built a bridge from one *Kilimanjaro* peak to the other together with Arthur Wilson.

First, let me briefly introduce you to the young climber who is addressing you.

The author

My name is Demian Nahuel Goos Bosco. I am half-German and half-Argentinian, and I originally wanted to become an archaeologist. The reason for this desire is probably hidden in the Roman background of my hometown *Mainz* in Germany, which, owing to the constant discoveries of historic nature, can also be described as a playground for archaeologists, and in the palaeontological riches of *Patagonia*, the southern region of Argentina, where I spent time while growing up.

However, when I read *Secret Codes* by Simon Singh at the age of 14, this desire suddenly had to move aside for a new one. By reading this book, I got to know Whitfield Diffie and Martin Hellman, who together developed the *Diffie–Hellman key exchange method*, which was published in their paper "On new directions in cryptography". I was fascinated not only by their story – the background of their friendship, the difficulties and conflicts that arose because of their work and how they faced them – but also by the cryptographic method itself. I was amazed by the simplicity and power of their algorithm. For instance, modern digital signatures are based on their method. It was a true game changer! And all that was obtained with simple computations!

I wanted to do that. I became enthusiastic, started creating my own cryptosystems and wanted to know what I had to do to work on these kinds of problems; after some research, I found that maths would be a good choice. Currently, I actually am a mathematician and although I do not exactly work on cryptography or anything somehow related to it, after many years of hard work, I got to the amazing moment of getting to know both of them (Whitfield Diffie and Martin Hellman) in person. An absolute highlight of my career! Pure excitement!

But how did this happen? What led to this incredible moment? To answer this question, I would like to go some steps back to the origins of this event.

The idea

Behind every big idea, there is a visionary, and the Heidelberg Laureate Forum is no exception. Born in December 1940, Klaus Tschira was what in Germany is called a *Kriegskind*, someone who was born during the barbaric Second World War, a term that calls to mind the traumatic circumstances under which these children were raised. However, traumatised is not an appropriate description for this man – quite the contrary!

If you look him up, you will probably find that he was one of the cofounders of SAP – one of the market leaders in software developing (but he would not appreciate this description). Being the forward-looking and passionate adventurer he was, a probable reaction would have been: "That happened so many years ago. I have done much more interesting things since I left SAP!" And indeed he did!

Once an entrepreneur, always an entrepreneur. He created the *Heidelberger Institut für Theoretische Studien*, an institution devoted to the research of natural sciences, mathematics and computer sciences. He also thought up the *Tschira Jugend Akademie*, whose mission is to connect young teenagers with biology and natural sciences. Art exhibitions and film festivals in which these dots are connected with mathematics and computer sciences are also part of his legacy.

In all of his projects and activities, there is always a fundamental idea: to create awareness of maths and computer science and their key role in everyday life. He understood that it is the scientist who must take the first step to do so, and this is what guided him throughout his work as a science patron. Two other beautiful examples are the *KlarText* Prize for communication in science, which is awarded to those young professionals who manage to describe their PhD thesis in a summary with such clarity that even schoolchildren can understand it, and his promotion of and collaboration with the *MS Wissenschaft*, a ship that, in 2010, sailed from city to city in Germany, bringing an interactive exhibition about the natural sciences to the people, particularly covering the environment, the dangers it faces and how science can contribute to avert this danger.

Last but not least, there is the *Heidelberg Laureate Forum Foundation*.

The Heidelberg Laureate Forum

As the name already suggests, the Heidelberg Laureate Forum takes places in this beautiful, charming, medieval city in the Bundesland of *Baden-Württemberg*. Heidelberg could not be a better host city for this event. It is not only a centre of research, knowledge and entrepreneurship but is also the hometown of the eldest university in Germany, a university that is connected to 56 Nobel Laureates and whose motto – *Semper Apertus* – seems to anticipate the nature of the forum and the spirit of its attendants. Inspiration is omnipresent during HLF and the city plays a vital role in it. Each morning, walking down the route from your hotel through the *Altstadt* and to the university, you feel like you are in an old village of a Grimm's tale, surrounded by picturesque medieval houses, the *Fachwerkhäuser*. At the end of the day, when it is getting dark, ambitious students from around the

globe meet their friends on the streets to spend time in one of the many *Kneipen* and pubs in the old town.

The concept of the Heidelberg Laureate Forum is simple: bring together the laureates of mathematics and computer science – the brilliant minds who were awarded the *Abel Prize*, the *Fields Medal*, the *ACM Turing Award*, the *ACM Prize in Computing* or the *Nevanlinna Prize* – and the next generation of outstanding young researchers and students in these two disciplines from around the world. In fact, more than 60 countries were represented in 2017, which also reflects the multicultural essence of the forum. While the young researchers are mainly PhD students and postdocs, undergrads can also apply to participate. These young researchers are selected by the award-winning organisations together with the *Heidelberg Institute for Theoretical Studies*, the *Mathematisches Forschungszentrum Oberwolfach*, the *Schloss Dagstuhl* and the *Heidelberg Laureate Forum Foundation*, whose Scientific Board makes the final decision. Nevertheless, these are not the only participants. Local researchers, scientific bloggers and representatives of science-related companies are invited to participate, and different media from around the world cover the event.

The forum can be partitioned into three groups of activities. In the first group, the plenary lectures, the laureates play the key role. Here, they can share their work, ideas and life stories through what I would call amazing and encouraging talks. In 2017, sixteen lectures were given and their contents could not have been more diverse. Manuel Blum shared his current research on consciousness in machines, Leslie Lamport introduced a modern structure of mathematical proofs and Vinton Cerf presented his (not so) futuristic work about an interplanetary internet. In the second group, which we could call scientific activities, the young researchers come to the fore. Postdocs can apply to organise a workshop in collaboration with a laureate, PhD students are invited to present their research during a poster session, and all young researchers can visit local scientific institutions and companies and, during a hot-topic discussion, interact with experts on a selected topic (this was quantum computing in 2017). Nevertheless, the most relevant part of the entire forum, which makes this event such a special and unforgettable moment in every attendant's career, is the third group: the social programme. Each activity is conceptualised with a particular love for detail to promote enriching, encouraging and inspiring conversations not only with the laureates but also with the other young researchers, journalists and others. On one day, you are taken on a long, relaxing boat trip on the *Neckar* among forests and castles and on the next day, you celebrate a private *Oktoberfest* with *Bier*, *Brezeln*, *Lederhosen* and a dancing *Volkstanzgruppe*. While one dinner is held within the *Speyer Museum of Technology* – chatting and dining with rockets, airplanes and even Da Vinci's inventions flying above you – the farewell dinner is held in the famous Heidelberg castle. What amazing scenery to conclude this spectacular week!

The conversations

It is very hard to describe the feelings that arise when talking to these renowned scientists. As computational complexity is one of my most significant and lively interests in mathematics, talking to Stephen Cook about the $\mathcal{P}$ vs. $\mathcal{NP}$ problem and discussing the motivation and inquisitiveness that led him to formalise the concepts (which laid the cornerstone for one of the most relevant problems in theoretical computer science) was by a long way one of the milestones of this week for me. Not every day do you get first-hand advice from Jeff Dean, the lead of Google's Artificial Intelligence Division, to start working on machine learning. Nor do you always get to discuss results in group theory with Efim Zelmanov – a mathematician who describes and explains his work in such a natural, intuitive and didactic way that it makes you feel that you actually understood each and every one of the explanations about his extremely abstract and intricate research.

Of course, it would be tempting to straightforwardly ask the laureates how they managed to be awarded these prizes so I could learn which path to trace but, obviously, there is no recipe for success. I actually overheard some comments such as: "If you want to be awarded such a prize, forget about its existence! None of us ever intended to get it." We could say that every path is like a *One–Time–Pad*: reuse is doomed to fail. Nonetheless, I certainly wanted to know what made these people stand out, how they think, how they faced their career and what drove them during their life. Basically, what do they have that others do not? What do they have in common?

To tap the full potential of this opportunity, I previously wrote a small booklet in which a spread is dedicated to each attending laureate, summarising the prizes they won and some biographic items I found interesting. The idea behind this was to check my booklet whenever I was talking with a laureate so I could ask more directed questions and avoid missing the chance to ask something I was curious about. The *book of the laureates* and the work put into it aroused much attention but, more importantly, it undoubtedly helped me achieve my goals.

Here, I put into writing the many observations I made, the most significant and valuable ideas the laureates shared with us and the most remarkable quotes I noted. I have made a particular effort to share these concepts as faithfully as possible and to properly reproduce the intention behind these words. I hope that I have succeeded.

Inspiring minds

The first thing that struck the eye was their humility. I was originally not quite sure how these conversations were going to go – formal and uptight or natural and casual – but the latter was clearly the case. The laureates were, as it turned out, very easy to approach and we changed from one topic to another as naturally as old friends. Not only in this sense could we notice their simplicity but also in many of the answers that they gave. To explain how the award changed his life, Alexei Efros, who generally always has a humorous reply ready, answered that he is still "that stupid, normal guy" that he was before he won the prize. When asked how it felt like to work on an idea he had many years ago when he was a young student – Manuel Blum chose his career because he wished to create *mechanical brains* and half a century later, he is working on consciousness in machines – he gave an unexpected answer: "It's scary. And very difficult." This was no less unexpected than Shigefumi Mori's answer to why he chose to study maths: "It was a mistake." This was not quite the answer you expect from someone who seems to have been predestined to do maths. He explained that he was a terribly shy kid, blushing whenever someone talked to him, and that he originally thought that maths was a way to avoid interacting that much with people. The current president of the *International Mathematical Union*, who attended this forum with the sole purpose of talking and interacting with people, concluded his anecdote with "I was wrong" and a likeable smile.

As a mathematician, I am aware of the fact that many deep-rooted prejudices people have about mathematicians are false. What I learned during HLF was something deeper. Part of the success of these laureates was precisely their ability to find a balance between their work and their lives. What I concluded is that each and every one of them has a very human aspect, and I got to know them as people and not as mathematicians. They did not allow their work to consume their lives; instead, they made their work part of their lives. Religion is a vital part of Frederick Brooks' life and he explained to me that being religious did not contradict his scientific career because he found his way to religion through critical reasoning. Martin Hellman says that meeting Whitfield Diffie was like "walking in a desert and coming across an oasis", and Diffie describes his wife as a "source of inspiration", without whom he would not have achieved anything. In particular, Hellman and Diffie's lives are a beautiful example of how important it is to get support in difficult times not only from your friends and family but also from the institution you work for and your colleagues. Religion, friendship, love – everyone found an equation of happiness in which their work is part of a whole, a variable in a multidimensional problem. They emphasised this throughout the week and underlined the importance of striving for happiness in all aspects of our lives. But what precisely makes us happy? Well, it is naturally something personal. Nobody can decide what fills our hearts with warmth, nor can anybody impose upon us what we are passionate about – not even ourselves! This is also valid in our research: we do not really decide which mountains we desire to conquer or which mountains have already conquered us. Madhu Sudan summed it up poetically with a Harry Potter reference: "The wand chooses the mathematician, not the other way around!" Vinton Cerf, whose hat seems to be a sympathetic homage to Hilbert himself, agreed: "We did not plan our careers. We did not plan the stuff we do!"

However, once we know the wand, the instruction is clear. "Do what you are passionate about!" Ivan Sutherland euphorically advised. Of course, I must say that this was not the only thing Sutherland told me with enthusiasm; I would say that in every word he shares, there is a conviction, certainty and passion. Without exception, they all work on those things that they love and we must do the same to succeed. We should not let anybody tell us otherwise – neither advisers nor colleagues – as Sutherland continued with a mathematical contradiction: "Do what you love – what makes you happy. And don't listen to anybody else. Not even to me!" After a brief reasoning, Bertrand Russell would probably have smiled.

The next tip was to be a detailed observer and to be open-minded when facing problems. What does this mean? I think a good example to visualise this concept is a real historical problem that the *United States Military* faced during World War II. They wanted to improve the performance of their air force and consulted the mathematician Abraham Wald. He observed the bombers returning from combat and noticed that some of their sections were far more damaged than others, which should intuitively induce us to improve the protection of these damaged sections. However, Wald did something different. He suggested that the areas that he observed to be undamaged should be protected, reasoning that if these sections were damaged by enemy fire, it would inevitably lead to the destruction of the aircraft, whereas any other damage would make it possible for the pilot to fly back to their base. To reach such a conclusion, it is important to think carefully, observe in detail, distrust your first intuition and reason about the implied consequences of each possible decision.

There is a beautiful painting of René Magritte from 1936 called *La clairvoyance*, which visualises precisely this

idea of observing more than the eye can see. It shows an artist observing an egg but painting an already born bird on the canvas.

Moreover, when the omnipresent Michael Atiyah, who, even at 88 years of age, was one of the most active, dynamic and compelling laureates, suggested that "we need to have quantum minds", he somehow told me to consider all possible solutions and not to discard anything until I find the best – until our mind has collapsed into one optimal observed state. There is an experiment that I usually do with my undergrads to show them how quickly we focus on one solution and discard a huge amount of alternate ideas. I ask them to tell me a prime number. After some answers, I ask them to tell me something that is not a prime number, and they intuitively answer with even numbers. At this point, I repeat my question and note that the Renaissance or even Lionel Messi are not prime numbers either and are indeed valid solutions to my question. Many students humorously complain that I cheated because I did not define a universal set or that I tricked them and that my first question directed their second answer – but then again, that is what I wanted to show them.

The laureates constantly encouraged us to work hard, to be persistent and to believe in our ideas. Michael Atiyah summed up this message very well when he said: "You don't need a million dollars, but energy, brains and perseverance." Stephen Smale gave a personal and very impressive illustration of this idea when he told me that he worked on the Poincaré conjecture but did not succeed at first. Later, when working on PDEs on manifolds, he thought that this could be a successful approach, which it eventually was. The idea of the necessity of failing to succeed was, in fact, hidden in many conversations. Manuel Blum commented with a bit of self-irony that he is "very good at getting his work rejected" and Robert Tarjan admitted that, even today, he frequently faces ups and downs in his work. Failure was explained not only to be an inevitable part of research and life in general but also a vital part of growth and success. But wait, there is more! Not only does failure seem to be quite necessary for success but so does a good bit of luck. At least, when presenting his intense way of working, Daniel Spielman gave me his real-life example that supports this idea. He told me that he came across the problem that eventually got him his award by pure chance: "What I did, I did it by accident."

This sounds disappointing at first glance, for what hope is left if we are subject to the capriciousness of chance? However, this is precisely a mistaken line of reasoning because luck is no random variable; we can shape our own luck and force it to go in certain directions. Efros shared with me two concepts in this context. The first is that luck is a consequence of attitude, which he illustrated by introducing two kinds of people. The first group consists of people who feel chased by bad luck, i.e. those who miss the bus, spill the coffee and forget their umbrellas on rainy days. Group two consists of those among us who seem to attract luck in an unbelievable way, i.e. those who find money precisely when they need it and catch the falling porcelain at the very last second. I am sure everybody knows someone who fits in each group. Clearly, both groups have a certain attitude toward everyday life, with the first group overlooking lucky moments and the latter not perceiving certain situations in which there is no luck at all. Efros then told me about a scientific experiment he read about, in which people from both groups were given the simple task of counting the number of triangles in a geometric figure in a magazine. The surprising result that the "lucky" ones were indeed quite faster had a simple explanation. Within the magazine, there was a page in which the solution to the problem was visualised and the people in the second group saw this and gave the printed number as their answer. Thus, people with a more positive attitude toward life are more likely to solve problems in a more effective way, perceiving alternate solutions and taking risks if need be. The second concept that Efros presented was luck as a consequence of hard work: "Luck helps the prepared mind. Great findings are never what you are looking for. But they are the result of hard work." Cerf also followed this direction when he motivated us to work hard to shape our luck: "You have to take your opportunities. Notice them and work. Work hard." How can you not follow this advice if it is given by someone who partially created the internet? Do not wait for the grass to grow.

Until now, everything I have summed up seems to be a pretty good recipe for how to achieve greatness in what you do, no matter what it is. Nevertheless, there is still something missing. After all, these guys have not solved just "something" but the big questions in mathematics and computer science.

Thus, how do you solve these big questions? How do you prove that $\mathcal{P}$ is strictly included in $\mathcal{NP}$ or that the Riemann Zeta-function has its zeros on the line $Re(z) = 0.5$? Well, there is one first step, which is quite simple: work on those problems and face them or, as Atiyah said: "Don't play with toys. Go for the big things!" In an unconnected conversation with Spielman, he unknowingly gave his own research as an example, when he told me that he started working on the Riemann hypothesis after finding a research article that somehow connected this problem to random matrices. Of course, facing these conjectures appears to be foolish. If so many brilliant minds have not found a solution, why would I? What could a fool like me achieve in comparison to them? Then again, as Hellman said: "Don't be afraid of being a fool. Only a fool goes where no one else goes." He is precisely a per-

son who achieved what he achieved because of being a "fool". Only a fool would question the intuitive notion that every statement within a consistent system of axioms can be proven to be either true or false, but Kurt Gödel did and thereby caused the foundations of mathematics to totter. Only a fool would question the natural idea that the position and momentum of a particle can be known with as much precision as desired, but Heisenberg's uncertainty principle puts a limit to precisely that precision.

I would like to finish with something Efros told me, which I found to be beautiful and inspiring as a strong, deep message:

> *"Look for ideas that change the world. Maybe you won't succeed. But you will have done a whole lot more than those who don't even try."*

Epilogue

After this week in Heidelberg, I needed some time to process the experience I had just had. All these lectures (where I barely talked at all), all these activities and conversations, all these thoughts and emotions – and all in just seven days! Back in Argentina, I wished to share my experiences with my friends, my peers and my undergrads; I wanted to make them part of this experience and really hoped to motivate someone to apply to participate in the HLF. With this in mind, I gave a talk at my university at the celebrations of the 50th anniversary of the Degree in Mathematics in *Rosario* (my hometown), which was the basis for this manuscript.

Now, the main purpose of this manuscript and the hope connected to it is to encourage more people to participate in this event. Whether you are a young scientist eligible for participation or a not-so-young scientist, I hope that the message is clear.

Hopefully, this work also shows new aspects on how mathematicians are and how they think and communicate. People frequently have a false image of mathematicians and I hope that this work provides a more realistic portrayal.

I would also like to honour the memory of Vladimir Voevodsky. He was one of the laureates who confirmed his participation but was missed during the event. One day after the forum was finished, he passed away. Whenever I hear someone talk about him, it is always with affection, respect for his achievements and admiration for his passionate way of working and communicating.

Acknowledgements

Finally, this work is also a way to express my gratitude to all those who somehow participated in this amazing experience:

- The laureates who participated in HLF 2017: all of those mentioned in this manuscript but of course also those laureates whom I did not mention, namely Martin Hairer, Tony Hoare, John Hopcroft, William Kahan, Joseph Sifakis, Richard Stearns and Leslie Valiant. The main reason for this is that during this week I did not have the opportunity to talk with you but I honestly wish to do so at another opportunity.
- The young researchers, with whom I also shared uncountably many incredible and encouraging conversations.
- The local scientists, journalists, bloggers and photographers.
- A special thanks to the organisers of the HLF for making this event even possible and letting me be part of it.
- And last but not least, my family and friends, who always support me in life.

Demian Nahuel Goos [demian@fceia.unr.edu.ar] is a half-German, half-Argentinian assistant instructor and PhD student in mathematics at Universidad Nacional de Rosario. Moreover, he is a teacher in German as a foreign language and a soccer referee. He enjoys doing 'artwork' about maths and mathematicians.