

# Interview with Fields Medalist Peter Scholze

Ulf Persson (Chalmers University of Technology, Göteborg, Sweden), Editor of the EMS Newsletter



**UP:** Let me start by asking you my standard, if somewhat silly, question I put to all new Fields medalists: were you surprised to get the medal?

PS: In view of all of the rumours I had been hearing for years beforehand that I was due to get it, it was in some sense not a surprise. But I actually felt pressurized by the rumours, so

when I was finally informed that I was getting the medal, I also felt some relief.

**So now you do not feel under any pressure to live up to the expectations that being a Fields medalist incurs?** I hope I can handle that.

**So how was the experience of being the centre of attention at Rio, in particular at the prize ceremony? By the way, was this the first ICM you have attended?**

No, I attended the one in Seoul as well. We had been warned about the media attention, and the number of e-mails we would receive.

**Was it that bad? I thought that the media attention was much stronger in South Korea than in Brazil.**

At the ceremony Figalli showed me that he had already got 200 e-mails on his phone.

**That is nothing. In Seoul the medalists got into the thousands as I recall, so they were very hard to get in touch with. Figalli got in touch with me very promptly.** He is very efficient.

**I certainly got that impression. By the way, did you know of the medalists already? When I was a young man, the names of the medalists were already familiar to me, but starting at the end of the last century most of them were unknown to me. I had heard of you before, but the other three names I had not heard of. Does it reflect that mathematics has become a much larger field and people are too caught up in their specialties to have an overview? Or is it just me being old and losing touch?**

Of course I knew of Venkatesh, after all he is a number theorist, and Figalli I had also met before, but I must admit that I was not familiar with the work of Birkar. Yes, I think mathematics is becoming a larger field.

**You left Rio early, was that because you wanted to get away from the attention?**

I had already attended a very nice meeting in Rio before the actual ICM, during which I got a good opportunity to see Rio itself, and at some point I wanted to return home. I am sorry you missed me.

**I am making up for it now. So, after this prelude let us be more systematic and start from the beginning. Your background. You grew up in Berlin?**

That is true.

**What do your parents do?**

My father is a physicist and my mother started out in computer science.

**So how and when did you discover mathematics? Did you have good teachers who were able to convey to you that there was such a thing as real mathematics, which is far more exciting than the standard mathematics offered at school?**

I had good teachers but I was also very much drawn to mathematics.

**Could you elaborate?**

When I was around fifteen or sixteen, I realised that Fermat's last theorem had been proven, and I tried to figure out what the proof was about -- elliptic curves, modular forms, etc. I did not understand anything -- I did not know what a matrix was, really -- but it was extremely fascinating.

**But how? Most students would never come across this, was it pointed out to you?**

I don't exactly recall, but it certainly played a role that I had good teachers, and that I met many like-minded students at the mathematical olympiads.

**I think this is natural for young burgeoning mathematicians, after all we are familiar with numbers from an early age, and may have played with them. But you were not discouraged by your lack of understanding?**

No, on the contrary. Because it all was very exciting and intrigued me a lot and I was dying to learn what it all really meant...

**...I can understand that. I had a similar experience, although at a much more elementary level, when I encountered Hardy's text book on Pure Mathematics. All those formulas with mystical symbols seemed like magic to me, and I thought that the way to happiness was through understanding and being familiar with them. Sorry, go on...**

...So I actually started a programme of systematically unravelling the mysteries.

**So one could say that you jumped into the middle of everything?**

I guess so.

**But other parts of mathematics, calculus for example. Did you not have that at your Gymnasium?**

Not yet. At the time I got involved, I was too young for that. So calculus, complex analysis, even algebra and all that I picked up on the side as part of the process of educating myself.

**When did you take your Abitur?**

At nineteen.

**That is the normal age. So you were not a prodigy in any formal sense. But you had participated in maths olympiads, I understand. When did you start?**

I started in Year 7 (equivalent to seventh grade) in some regional olympiads. Later on I was able to reach the international olympiads.

**You did very well, did you not, repeatedly winning gold medals?**

That is true. I did not think that I was going to be able to do so, I thought that those who had won them before were much better than me, but to my surprise I was able to do as well.

**So what is your opinion on those spectacles, are they a good thing or not?**

I am not quite as negative as Rapoport is, but I can see the danger that people are diverted by obtuse and contrived combinatorial problems.

**But did it give you a lot of self-confidence?**

I guess so.

**People who are good at olympiad problems usually are technically strong, or at least have the potential to become so, and as a research mathematician, especially if you want to be reasonably self-sufficient, such skills are invaluable, although not sufficient.**

Yes. Mathematics is more than solving problems (let alone within time limits), and what excites me is getting a global understanding.

**You mean, as actual designation implies, olympiads reduce mathematics to a form of athletics? What is your feeling towards competition in mathematics? As you have been very successful one may think that you would thrive on it.**

First, as to the math olympiads, the major thing I got out of them was actually the social aspect. It was great meeting people all over the world with whom you could share interests...

**...that was certainly my experience as well. Sorry, go on...**

And as to the competitive aspect, as I told you already, what I get out of mathematics is the thrill of understanding, being the best is not necessarily important to me at all. On the other hand, I think that a certain element of competition is actually good. It pushes us to do our best.

**To keep us on our toes, you mean?**

Yes, if you prefer.

**In chess they say the whole point of it is to see who is the best. Chess without competition would be meaningless. Hence, in chess you can rank people fairly accurately through an algorithm we need not be concerned with. If there were a similar system in mathematics, hiring would be greatly facilitated, but in mathematics there is no meaningful way you can rank people linearly; mathematics is too large and multifarious for that. However, in recent years there is much emphasis on various indices: citation indices, h-indices and whatever. The bureaucrats love it because it is of course objective and**

*can be computed with no regard whatsoever to the contents. All you need is to draw a graph; the nodes themselves mean nothing. It makes a travesty of mathematical competition.*

I see what you are driving at. I know some very good people who, by the time they had established themselves as a leading mathematician, still had an h-index of maybe two.

*In the past, people were judged by their work as such, now I fear there are just too many people, so it is no longer feasible. Is there too much mathematics being published? After all, most papers written are not published for their intrinsic interest, but because people need to get ahead and improve their lists of publications and citations. Although, most papers are lucky if they are at least read by the referees.*

I have no opinion on that matter. In my own field I do not think this is a very serious problem.

*Now let us get back. There you were at nineteen, quite advanced in mathematics, what did you do?*

At the time I was wondering whether I should study in the US, but somehow I wanted to stay in Germany, yet I wanted to leave my home town Berlin. I was in contact with Altmann, who was an algebraic geometer and he suggested Bonn and mentioned Rapoport.

*Rapoport was of course a bit sceptical about you, your performance at the olympiads did not cut any ice with him, and suspecting that you might be a mere charlatan he subjected you to a third-degree interrogation, from what I gather. How did you experience being on the receiving end, so to speak? Did it take long for you to convince him that you were not a charlatan but very much the real thing?*

I have no particular memory of being subjected to very harsh treatment. But Rapoport is a very serious man you know.

*I know.*

Do you have any more questions?

*I am loath to keep you, but let me just put a few more. I understand that you became enrolled as a graduate student right away.*

No. Rapoport insisted that I go through the regular system, taking undergraduate maths courses. But he did give me a few problems to think about.

*He was no doubt concerned about your general mathematical culture. Did you resent it?*

No! The only thing that I didn't enjoy much at the time were the experiments in physics.

*At what age did you finish your thesis?*

At twenty-four.

*That is not exceptional, but I suspect that you did some exceptional things before that.*

I had found a simple way to compute the  $L$ -function of the modular curve, at bad places. A little later I realised that one could use these techniques to simplify a key step in the proof of the local Langlands correspondence for  $GL_n$ , for which I got a Clay scholarship.

*And made your reputation outside Bonn, I surmise.*

I suppose so.

*Do you co-operate with people now, or is this really impossible to do so profitably with someone not at your level? It is hard to think of Gauss working with someone else in mathematics (with physics it was different as we know).*

Of course I do. I like to share my ideas with colleagues.

*Mathematics is very much a social thing.*

It certainly is.

*So, let me formulate a moral question for you. If you discovered that  $A$  implied the Riemann hypothesis, say, would you keep quiet and wait until someone proved  $A$  and then step in because it is the one who puts in the last brick in an edifice who usually gets the major credit?*

This sounds like some contrived speculative question that does not interest me.

*Point well taken. What about your collaboration with your advisor Rapoport. Did it mainly concern suggestions on his part, or where you engaged in technicalities?*

It was mostly one of general suggestions.

*So you are self-sufficient in this respect. By the way, do you read a lot? And if so, do you read systematically, from cover to cover, or do you skim looking for the meat?*

I read a lot. Some books I do read from cover to cover, especially when I am trying to learn the basics in a new field, but otherwise I often skim an article to find the information I care about.

*You seem very focused on mathematics, do you have any other interests? When you were first pointed out to me proudly by Rapoport in a Bonn Cafeteria some years ago you were in a cast. Are you a skier?*

Oh no, this was just a stupid accident and had nothing to do with any athletic activity, and besides if I ski I do cross-country. I pretty much do all my thinking in mathematics.

*Let us change tack. What you do in mathematics is mainly a matter of taste, and your taste was formed early on. Are there some parts of mathematics which you find distasteful?*

[long silence]. Well, there are certainly parts of mathematics, and maybe also styles of proofs, that I like more than others.

*Because even if logically impeccable it admits no global understanding but is simply thrown at you?*

Proofs should be based on an idea, and the methods used compatible with the idea somehow.

*It reminds me of Grothendieck who claimed that proofs should be natural and part of an overarching structure. He hated tricks and ad hoc intrusions. Proofs should be instructive and explaining, not just formal verifications. That goes without saying.*

*You mentioned that your mother started out in computer science. Do you program? This is something which nowadays would come naturally to mathematically inclined children, but was not available when I was a child. And if so how would you compare programming to mathematics?*

Before I encountered mathematics at fifteen I programmed. I even designed computer games. But it was all pretty childish and when I became caught up in mathematics I stopped doing it and have never taken it up again. I cannot make any comparisons, because my programming experience is that of a child, and that of mathematics of a mature professional adult.

*I would say that programming is relaxation, akin to crossword puzzles, although I personally have no interest in the latter. You always know that you are going to succeed eventually, and you never get stuck in the same way as in mathematics, as you have a continual interaction with the computer and can engage in trial and error, in a way which is not available to you in mathematics. But you also discover how many mistakes you*

*make, and would the same not be true of maths papers? Most are fixable of course but there may be serious mistakes which one will never discover, as the results are not interesting and no one really cares about them (maybe not even the authors!). But of course if someone claims to have proved an important hypothesis, the proof is subjected to relentless scrutiny and more often than not serious gaps are discovered: in some cases even leading to a total collapse of the approach. This also being the case with highly regarded mathematicians to boot.*

Well, I guess that is somewhat true, but I am still very confident in the mathematical literature, certainly in the parts I know well. By the way, how much longer do you need? I do not want to miss the next lecture.

*Do not worry, you will be sure to catch up quickly. And an interview like this is not based on a prepared list of questions to be ticked off; like all inquiries it is open-ended in the sense that every response suggests a new previously unthought of question.*

That may be very well, but at this stage I prefer closed to open.

*In that case I do not want to keep you any longer, thank you very much for your time.*

*Ulf Persson is on the Editorial Board of the EMS Newsletter. His photo and CV can be found in previous Newsletter issues.*