ERME Column

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ERME Thematic Working Groups

The European Society for Research in Mathematics Education (ERME), holds a biennial conference (CERME), in which research is presented and discussed in Thematic Working Groups (TWG). We continue the initiative of introducing the working groups, which we began in the September 2017 issue, focusing on ways in which European research in the field of mathematics education may be interesting or relevant for research mathematicians. Our aim is to extend the ERME community with new participants, who may benefit from hearing about research methods and findings and who may contribute to future CERMEs.

Introducing CERME Thematic Working Group 8 – Affect and Teaching and Learning of Mathematics

Thematic Working Group rationale

We would like to begin with a quote by G. Pólya, a Hungarian mathematician, who wrote *How to Solve It* (1945) – his seminal work on problem solving:

> *"Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. … If he [the teacher] fills his allotted time with drilling his students in routine operations, he kills their interest…"* (Pólya, 1945, p. V)

This quotation demonstrates that affective concepts such as emotions and interest were identified as important antecedents of mathematical thinking and problem solving even in the early days of mathematics education. Moreover, improving students' positive affective experience was suggested as essential for teaching mathematics. However, despite the importance of affective factors for learning and teaching mathematics, and for problem solving and mathematical thinking, it took a long time until mathematics educators began to focus on affect as a research domain. A considerable attempt at systematising the field was done by Douglas McLeod in the early 1990s (1992). He proposed important characteristics of affect, such as stability and intensity of affective variables, and made a comprehensive review of research results on affect, pointing out the lack of theories in research on affect. Since then, the research on affect has received more and more attention and has been featured in a number of keynotes at conferences and in high-impact journals in mathematics education.

Discussions about the importance of affect in TWG8 (founded in 2003) have contributed significantly to our understanding of affect and its role in teaching and learning mathematics. McLeod (1992) identified affect as including a wide range of concepts including beliefs, attitudes, values, goals, needs, motivation, identity, self-esteem and emotions. Discussions on similarities and differences between these affective concepts is an important part of work of TWG8 at each conference. Because of conceptual overlap between affective variables and origins of affective concepts in different theories, clarifying the relations of affective constructs to each other is demanding work. Stability (stable trait vs. unstable state), intensity (high vs. low), or valence (positive, negative, neutral) are the main characteristics that can be assigned to each affective variable. Another topic of high relevance in TWG8 is the evaluation of affective measures. We have explored, and continue to explore, the advantages and shortcomings of traditional approaches, such as questionnaires and student interviews, and innovative approaches, such as eye-tracking, measures of heart rate or skin conductance as indicators of students' affect.

Most relevant for the readers of this newsletter might be the relation between affect, mathematical thinking, and the learning and teaching of mathematics. Recent research has demonstrated that there is a feedback loop between affect and mathematical thinking, and has stressed the importance of teachers and parents in the development of students' emotions, motivation and other affective concepts.

Affect in early childhood and in schools

We do not know much about the development of affect in early childhood and in primary school. Findings from research on anxiety indicate that some young children report anxiety around mathematics even at the very start of formal schooling. A negative relationship between maths anxiety and mathematical performance has been identified in students as early as first grade. Children with early mathematical anxiety receive lower grades, and lower grades in turn increase their mathematical anxiety (Carey et al., 2017). This can follow students all the way to upper secondary school, where low self-confidence, low enjoyment and high levels of boredom have been identified as reasons for not continuing with mathematics in university.

One of the factors that has been shown to contribute to the development of anxiety is students' experience with teachers and teaching methods. Although early research on mathematical affect was more concerned with identifying the sources of such anxiety, more recent research has been looking at ways to mitigate anxiety. Such research has shown that teachers' positive affect toward mathematics increases students' positive affect

towards mathematics. For example, teachers' enthusiasm and enjoyment of teaching mathematics affected students' interest and enjoyment of mathematics in lower secondary school. Likewise, prompting students to develop, compare, and contrast multiple solutions for a problem was shown to increase students' enjoyment, autonomy and perceived competency during mathematics classes, and improved their interest in mathematics.

Affect in universities

The role of affect in tertiary education is of great interest in TWG8. We all know that students struggle in their first-year mathematics courses and, as a result, many students drop out of mathematics. Many of these students are accustomed to being successful in mathematics, and to struggle is unexpected for them and creates an onset of negative affect – "*… for the first two years I was in a permanent state of anxiety and distress"; "I think that my experience at the Bachelor in mathematics left me with less confidence in my ability to study"; "I remember [of that period] just a lot of tears"* (Di Martino & Gregorio, 2017). One reason for these affective difficulties is a big difference between mathematics in upper secondary and tertiary education. Whereas in upper secondary school mathematics is a subject in which students apply mathematical procedures and solve word problems, in university mathematics is a subject of formal language, justification and proof. Consequently, many students do not recognise their favourite school subject and develop motivational problems.

In order to support students through this transition, many universities have started various intervention programmes. These programmes often begin with preparation courses that aim at refreshing students' prior knowledge and introducing them to the main proof techniques. Another intervention is the introduction of learning centres, which offer students the opportunity to share their thoughts while working on lectures and sample problems under the guidance of teaching assistants. Although these types of interventions reduce the failure and dropout rates of students in first year mathematics courses, we do not know much about the effects of such programmes on students' emotions and motivation. Research on instructional practices that support students' perceived competence and autonomy, increase students' motivational beliefs and improve their positive emotions, is still developing. However, an approach which has shown promise is to focus on the individual development of students' understanding ("I improved my knowledge in the last week") instead of their performance ("I got a higher score last week").

In this overview, we have offered a brief description of the development of affect and have focused on topics that have been discussed in TWG8 in recent years. We welcome in our group mathematicians, teacher educators and researchers to share our views on affect and its relation to the teaching and learning of mathematics at CERME 12 in 2021.

References

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Peter Liljedahl is a professor of mathematics education in the Faculty of Education at Simon Fraser University. He is the former president of the International Group for the Psychology of Mathematics Education (PME), and the current president of the Canadian Mathematics Education Study Group (CMESG), as well as a senior editor for the International Journal of Science and Mathematics Education (IJSME). Peter is a former high school mathematics teacher who has kept his research interest and activities close to the classroom. He consults regularly with teachers, schools, school districts and Ministries of Education on issues of teaching and learning, assessment and numeracy.

Stanislaw Schukajlow is a professor of mathematics education at the University of Münster, Germany. He has co-edited special issues on emotions and motivation, modelling and word problems in ZDM Mathematics Education. His research interests include motivation, emotions, strategies, teaching methods for modelling problems in schools and development of pre-service teachers' affect in universities.

Jason Cooper is an associate staff scientist at the Weizmann Institute's Department of Science Teaching. His research concerns various aspects of teacher knowledge, including roles of advanced mathematical knowledge in teaching mathematics and contributions of research mathematicians to the professional development of mathematics teachers.