Working from Home. ~~2 Months~~ 4 Months and Still Counting…

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Dear friends and colleagues,

These days most of us are working from home and this seems to be a unique experience. We thought of writing an article about this topic for the Newsletter of the European Mathematical Society.

So a couple of questions for you:

One way or another, some of us replace school teachers for our kids from kindergarten through to university (tough job, right?) You have probably invented some nice recipes for how to explain mathematics and science in general to non-specialists that you are willing to share. Or you have found some resources that you have used successfully and would like to advertise. Please do!

The other way around: your nearest and dearest can now see you working. Maybe you have finally taken the time to explain what your job really is? What kind of science you do and why. Maybe even convinced someone that watching a seminar on your TV in the living room is more interesting than cartoons? Or just some interesting discussion happened unexpectedly? Or even better, a funny episode? Please share!

Impatiently waiting for your stories.
Yours, Vladimir

My letter, 23/04/2020 (Vladimir Salnikov)

This is the e-mail that I wrote to a rather large list of people to whom I usually make scientific announcements. It sort of speaks for itself: I realised that we were (and still are) crossing some historic period, and I was curious about how other people are handling it.

I received several replies, from which we chose some that people were ready to share with a broader audience, hence this article.

Actually, I received a lot of material, so we will continue in the next issue of the EMS Newsletter. And since we now have no idea how the situation will evolve, I will be attaching dates to be able to fit the stories into the context. In this issue we will address mostly the family aspect of the story, and the second part will be about everything "online". I do hope it will not become a regular section, though. And I will start with my own short anecdote.

My son Miron has just turned 4 years old, and he actually motivated me to write the above letter by showing me how I work. He glued together two sheets of A4 paper, drew some sort of rectangle on one of them and small rectangles on the other one. You have guessed it: he said it was his laptop. Then the following happened (I just watched, with no interference). He sat on the sofa in the living room, opened his "laptop" and said, "I am going to work now". He pressed the "buttons" for a couple of minutes, then said "No, it is too noisy here", he closed the "laptop" and went to his bedroom. He closed the door and stayed there for about 3 minutes (I do not know what he was doing, but from the sound I guess he was still typing). He came back, his "laptop" under his arm and said, "I was working, but now I am finished", and put it aside. Well, I recognised myself … did you?

Laura Schaposnik, letter from 28/04/2020

Being pregnant and thus at high risk, we started self-isolating before any official advice was given here in Chicago. Hence, now being in week 6 of house confinement, we've developed some favourite ways in which we're teaching our 20 month-old son about numbers and letters.

Our day starts at 7 am, and after some light reading we offer little Nikolay some "pasas de uva": that's when he runs to the table, knowing there will be 5 raisins ready for him, displayed in a line which will get shorter as he eats. Each time one is gone, we count backwards… by now, the little habit has grown into him tidying all his food into lines and making as if he were counting the objects. Maybe he's counting them in his head – the sounds he makes, albeit not English or Spanish numbers, are certainly precise and get repeated every time he is counting!

Whilst most of our books are not about counting – in fact, we don't have any about it! – we have turned most of them into letters and counting games… this is how

we end up in an "arándanos" party, counting backwards from 8 most lunchtimes! The newest problem on this front is that Nikolay loves hearing that "no hay ningún arándano más!" and thus munches 8 blueberries within seconds.

We have also decided to make certain actions be accompanied by counting every time they occur – when waiting to cross a street, heating something up in the microwave, or preparing a piece of toast, we'll always count to 10 (in Spanish) and this has by now led to Nikolay "counting" by himself (and moving his hands adding random fingers to the count) when these events happen.

One of the things we love the most from these last weeks is that he knows what "one", "two" and "many" are (with one finger, two fingers, or ten fingers) and you can now ask him how many of something he wants… which is usually either none, or many many more!

Alessandra Frabetti, letters from 27/04/2020 and 29/04/2020

I submit three contributions :-)

1) My son is seventeen, he is finishing the Lycée[1] and fortunately he does not need my help anymore to do his homework (if he ever needed it), he is fully autonomous. But I do have a funny story to tell about replacing cartoons with online math seminars. The funny side is that I didn't force my son to, didn't even suggest it, rather the opposite!

My son wants to study informatics next year, and he is a great fan of online lectures on his favorite topics, in particular those offered by MIT. A few evenings ago, at dinner, he explained to me the so-called samples representation of polynomials, that I had never come across, in particular its good behaviour with respect to the basic operations among polynomials. He was so excited that he finally proposed my husband and I immediately watch, on the laptop, the lecture "Divide & Conquer: FFT"[2] by Professor Erik Demaine of the Department of Electrical Engineering & Computer Science at MIT.

And that's how the whole family watched an MIT online course about the fast Fourier transform during supper!

An unusual, interesting and amusing supper, indeed!

2) I would like to suggest a trick for encouraging children to compute sums and products, and also for opening the wonderful world of abstraction up to them. I experienced it a long time ago (when my son was six years old, in 2008), but it adapts perfectly to the current confinement situation and I would surely do it again if I had young children around.

Every parent knows that answering "what is three times eight?" is of no interest to a child in primary school.

But if you first tell the child what a two-by-two matrix is a mysterious box with four numbers, from which one can extract so much information! – especially if you spice it up by saying that it is a topic of your lectures at the first year of university, and you tell them the rule to compute its determinant, I'm pretty sure that any child will play at "Let's Compute Two-by-two Determinants" for quite a while!

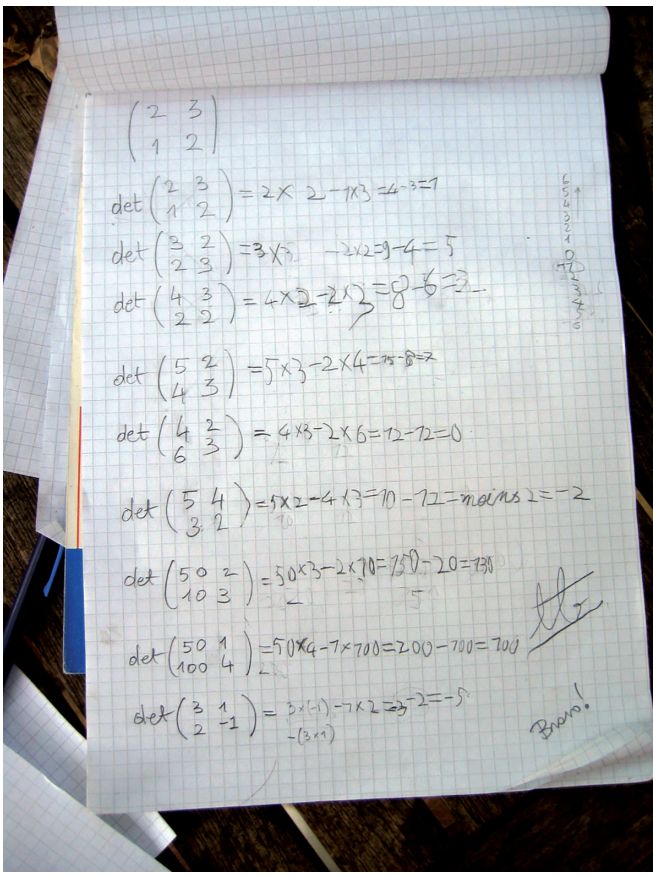
And if the confinement lasts longer, why not try computing the product of two-by-two matrices? And the inverse?

I think any operation with matrices is really fun!

With this trick, I once kept my son and a friend, both six years old, busy for half an afternoon. I started with matrices with small numbers (1 to 5), and after half an hour the children asked me for bigger numbers because it was "too simple". I had to insert some 50 and 100 to make it fun enough! I attach two pictures to show its success.

[1] High school in France, age from 15 to 18.

[2] https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-046j-design-and-analysis-of-algorithms-spring-2015/lecture-videos/lecture-3-divide-conquer-fft/



In the next days they asked me for more, and I moved to the product and to the inverse of matrices.
A short time after, I opened the new fantastic world of "Solving Equations in the Unknown x": find x knowing that $x+2=3$, or $2x=6$, or even $2x+1=5$. They truly adored it!

Can you predict the answer when I asked my son, during a long annoying trip by car, "Can you find all x such that x times x is equal to x?" He found three solutions, yet his answer was correct: "0, 1 and infinity"!

And can you predict the reaction when I proposed enlarging the set of numbers to some totally unreal, imaginary numbers (the complex numbers of course)? I think my son was around 8 when I dared.

I saw his full attention on this new trick, his ears wide open and his eyes shining :-) Try it!

3) Finally, in the same age range, my son simply fell in love with the representation of base-2 numbers on one's fingers. Finger up, finger down, he learned to count in base-2 at the speed of light! The game is then to be the quickest to count up to 31 without making any mistakes. And even higher, with the second hand. Of course he always won!

And now he knows the powers of two up to… well, I don't know, because I don't know so many powers of two ;-)

Vladimir Salnikov
Dear Alessandra!

Thanks for sharing the stories – I liked them a lot.
I think I'll give the determinants a try. Eventually explaining areas as well (we are geometers after all). Daria is precisely 6 now, and is obviously bored by the usual counting stuff. I guess that would bring some excitement to my kindergarten self-isolation :-)

Greetings to the family!
Ciao, Vladimir

Alessandra Frabetti
Ah, Daria is at the perfect age! :-)

About areas, sure! I had two more experiences with Adrien, inspired by areas and geometry, still around 6–8 years old.

The second one can be done with even younger children if you skip the explanations on the dimension and on fractions. I presented it to some *Fête de la Science* in Lyon and in Paris, for children of the Ecole Maternelle.

1) The game "What's the Dimension?"
First, I asked Adrien to draw segments of length 1, 2, 3, etc. and I asked him what the length was (rhetorical question).

Then I asked him to draw squares of side 1, 2, 3, and I asked him what the area was (the number of little squares).

Then I asked him to draw cubes of side 1, 2, 3, and I asked him what the volume was (the number of little cubes).

Finally, I drew a table

side	area	volume
1	1	1
2	4	8
3	9	27

and I asked him to guess the line with side N (any length): the exposant D of the power N^D is called the 'dimension' of the drawn object. The dimension of an object counts the number of important 'thicknesses' of the object.

One can remark that the 'boundary' or 'contour' of a square (of dim 2) is composed of segments (of dim 1), and that the 'boundary' of a cube (dim 3) is composed of squares (of dim 2)!

You can also push to the 'boundary' of a segment (of dim 1), which are two points: let's call them of dimension 0!

To go from dimension D to dimension D-1 is equivalent to slimming down one thickness until it becomes 'negligible', insignificant, with respect to the others.

Then we played "What's the Dimension?" everywhere!

For instance, at lunch: the tablecloth is clearly of dimension 2, and we can view a very thin plate as a 2-dim object, and a very thick one as a 3-dim one. What about an empty glass? What is the boundary of a glass?

What about the dimension of the water inside a glass full of water?

What is the dimension of an apple? That of a crêpe (a think pancake)? That of a single piece of spaghetti? What is the dimension of a cat?

A joke: a material is 'liquid' if it takes the form of its container. My brother showed me that a cat is liquid! In fact, as it is well known, it takes on the shape of any container :-)

2) The drawing "Mickey Fractal"

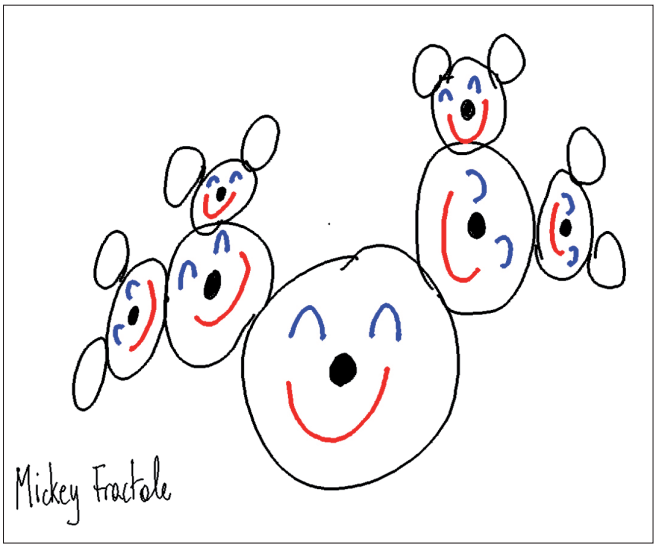
Once the 'integer dimension' is well established in the child's mind, and the child is comfortable with fractions, one can go further. Usual objects have integral dimension, but there exist objects with non-integer dimension: fractals!

The trick of fractals is that they are plounged in a D-dimensional space in a crazy way, so that their dimension is not D (they are sensibly thinner than the whole space) but it is bigger than D-1 because they occupy too much space by zigzagging here and there at any point: none of the D-space's thicknesses can be neglected!

For instance, Norway's coast full of fjords is a fractal coast on the Earth surface.

In 2007 I invented a Christmas card called the Fractal Christmas Tree for Adrien's class, to be constructed with two sheets of paper with different colours, a pencil, scissors and glue. I then proposed it to some Festival of Science and it is now well known, you can find the instructions to build it online, cf. at https://eduscol.education.fr/maths/actualites/actualites/article/carte-de-voeu-fractale.html

I then invented another easy fractal for very small children, in order to understand the procedure of 'doing the same action in a smaller size': Mickey Fractal.



Start with Mickey's face: a big circle with a nose, two eyes and a (smiling) mouth, together with two big round ears.

Then imagine that each of Mickey's ears becomes a smaller Mickey: draw again the nose, the eyes, the mouth and the two ears, and so forth, cf. the joint drawing.

Of course you can imagine plenty of easy fractals of this type, and there are thousands of pictures online.

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